COLLEGE OF THE VIRGIN ISLANDS

THE EFFECTIVENESS OF THE CUISENAIRE RODS ON THE INSTRUCTION OF EDUCABLE MENTALLY RETARDED STUDENTS

A THESIS SUBMITTED TO
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CHAPTER I

BACKGROUND

The role of educators has been for some time to teach students, whom they come in contact with, the necessary skills that they will need in order to function within our society. Among the goals of education, one of the most important goals is the transition of knowledge to these youngsters. The knowledge should not be just mere facts and information. Education also has the goal of instilling a desire to learn and to achieve in those students who will be the future leaders in our society. With this in mind, it is hoped that they not only are able to function within our society but to contribute in a positive way to the further development and improvement of the world in which we live. Therefore, the best methods for teaching certain skills so that they may be retained for further use in society are being constantly searched for and analyzed in terms of their effectiveness in learning.

It is a common belief that every child has the right to an education. The educable mentally retarded child is no

exception. He deserves an education also. He was unable to fit into the system of education as we know it. However, due to the passage of Public Law 94-142, every handicapped child has been given the right to an education. The child is no longer forced to change so that he fits into the school system; the school system must change so that it fits into the characteristics of the child. The school system must provide the necessary environment along with the materials to insure that these students develop to their full potential. If the school system does not comply with the law, it can be held liable.

Educable mentally retarded students are considered as part of the group called handicapped or exceptional children. They need to be educated so that they can function within our society and not be considered inferior. One of the most important areas to acquire enough understanding is in the area of mathematics. A knowledge of mathematics is needed in practically every facet of today's society. Mathematics helps us to know the number of days in a month, the amount of any object we may have at a set time, and the amount of money to spend, invest, or save. Not only is the knowledge of mathematics helpful, it has set purposes.

Berger lists the purposes of mathematics as follows:

ulative devices enable the student to actually operate, hold, and really investigate what is placed before him to conceptualize. According to Piaget, learning proceeds through stages.[1] One must start with the concrete level, then to semi-concrete, then to abstract. Here again, we see the need for concrete or manipulative objects for concept learning.

There has not been much research done on the skills of educable mentally retarded students in the area of mathematics. However, the research that has been done shows that certain characteristics are found in the educable mentally retarded students that are typical of that population in terms of mathematics. Glennon summarizes these characteristics as follows:

1. They are retarded in the area of arithmetic vocabulary.
2. They are inferior to normal students in the ability to solve abstract verbal problems.
3. They are better at solving concrete problems than at solving abstract problems.
4. They have less understanding of the processes to be used in a problem situation and are more apt to guess at processes than normal students.
5. They are more careless than normal students at their work, use more immature processes, and make more technical errors.

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[1]Jean Piaget, "How Children Form Mathematical Concepts", Scientific American 189 (November 1953), p. 74.

6. They are less successful in differentiating extraneous material from needed arithmetical facts than normal children.

7. They do equally well with word problems and mechanical operations if the instruction is meaningful to them.

8. They have little concept of time and sequence.

9. They do better with addition and subtraction and need more emphasis on multiplication and division.

10. Arithmetic readiness is even more important to the educable mentally retarded students than to normal students.[1]

With these characteristics, it has been found that this group of students seem to work up to mental age expectancy in mathematical fundamentals. However, this does not occur in arithmetic reasoning in which there is reading and problem-solving. The educable mentally retarded student does develop quantitative concepts in the same order and stages as normal children do. Although they develop these concepts, they are developed later. The concepts are acquired through teaching and maturation. When students are drilled to perform advanced Piagetian-type quantitative tasks, they may appear to have the skills, but they do not really understand the concepts involved. Understanding of the concepts comes

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[1]Vincent Glennon and Leroy Callahan, Elementary School Mathematics: A Guide to Current Research, 3d ed. (Washington, D.C.: Association for Supervision and Curriculum Development, NEA, 1968), p.45.

at a later time after adequate intellectual growth has taken place.[1] According to a number of researchers, concrete materials for teaching mathematics seems to work well with these students. In a similar fashion, it seems as though they learn a particular concept much better when it is presented in the concrete mode. An example of a concept would be addition. In the concrete mode, the child is given two blocks and then two more blocks and asked how many there are all together. The concrete mode is represented through the blocks. The learning style of the educable mentally retarded student is one that encourages him to actually see, hold, and manipulate the material at hand so that it will help him understand the concept. They learn through concrete representation of the subject matter.

Cuisenaire rods emphasize this learning style. They are colored rods that range from one centimeter to ten centimeters long. The student is able to visualize and manipulate the rods. He is able to internalize the concept and gain a better understanding of mathematics by using the rods. These rods were named after Georges Cuisenaire, who invented them. He was a schoolmaster in Belgium. It was because one of his students had problems in understanding mathematics that he started experimenting with pieces of wood. The pieces

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[1]Lloyd Dunn, ed., Exceptional Children in the Schools, (New York: Holt, Rinehart, and Winston, Inc. 1973), p. 148.

were painted various shades of the primary colors, one in black, and the smallest size was not painted. The students in his classroom experimented with them. They soon gained confidence and did mathematical operations with them. Caleb Gattegno was the person responsible for making the rods popular by promoting them abroad. According to Caleb Gattegno, "Color is a factor that is accessible to the minds of almost all humans. Its shades and contrasts can act as a sign to substitute for the abstract notion that it is proposed to attain."[1] It seems as though color would catch anyone's eyes. After this attention is obtained, maybe some learning can be nurtured. The colors for the Cuisenaire rods are based on the following system:

1. All the multiples of two contain the color red, (2cm- red, 4cm- purple, 8cm- brown)

2. All the multiples of three contain the color blue, (3cm- light blue, 6cm- dark green, 9cm- blue)

3. All the multiples of five contain the color yellow, (5cm- yellow, 10cm- orange)

4. Number one was uncolored; number seven was considered the outcast and was colored black.

The Cuisenaire rods focus on the belief that the student sees relationships with the rods in terms of mathematics. These relationships are formulated by the students

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[1]William Ewbank, "The Use of Color for Teaching Mathematics," Arithmetic Teacher, 26 (September 1978): 53.

based on the activities engaged in with the use of the rods. According to the producers of the Cuisenaire rods, the rods capture the attention of students, and this enables students to focus on the task at hand. The philosophy of Cuisenaire rods is one of active teaching. It focuses on seeing, doing, reckoning, understanding, and verification. Based on this philosophy, the student is engaged in manipulation of a concrete set of object, namely the Cuisenaire rods, to work out mathematical operations.

PROBLEM STATEMENT

The question of how to teach the educable mentally retarded has been discussed often. Most researchers feel that they learn best when the subject matter is presented in a concrete manner. In the area of mathematics, the educable mentally retarded have a difficult time conceptualizing the ideas involved. They learn at a slower pace then other students. The particular type of material to be investigated with the educable mentally retarded is the Cuisenaire rods. This material is concrete in its presentation and is used in the area of mathematics. The question of how to best teach the educable mentally retarded mathematics needs to be addressed. How effective are the Cuisenaire rods in helping educable mentally retarded students grasp the concepts of addition and subtraction of one-and two-digit whole numbers? The study will, therefore, look at the effectiveness of Cuisenaire rods on educable mentally retarded students in the area of addition and subtraction of whole numbers (one- and two-digit numbers).

HYPOTHESIS

There will be a significant difference in the achievement of the educable mentally retarded students using the Cuisenaire rods as compared to those using the traditional method.

DEFINITION OF TERMS

1. Educable Mentally Retarded Students A definition as stated by Van Osdol and Shane in An Introduction to Exceptional Children:

> Mental retardation refers to significantly subaverage general intellectual functioning existing concurrently with deficits in adaptive behavior and manifested during the developmental period.[1]

2. Concrete Materials Those materials which one is able to see, feel, and manipulate.

3. Cuisenaire Rods Colored rods which range in size from one centimeter to ten centimeters long. They are used to improve computational skills and increase mathematical understandings.

4. Traditional Method The use of specified textbook and workbook in teaching mathematics along with the use of drill exercises on the chalkboard with the use of concrete materials when needed.

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[1]William R. Van Osdol and Don G. Shane, An Introduction to Exceptional Children, (Iowa: Wm. C. Brown Company, 1974), p. 57.

LIMITATIONS

The study will be limited to a group of eighteen primary educable mentally retarded students. It will be conducted at two elementary schools on the island of St. Thomas. There will be ten students in the experimental group and eight students in the control group.

SIGNIFICANCE

This study will look at a particular material, the Cuisenaire rods, in terms of its effectiveness with educable mentally retarded students. It is hoped that the study will be of some use to teachers of educable mentally retarded students in their search for appropriate materials for their students.

CHAPTER II

REVIEW OF LITERATURE

In the field of education there is a constant concern about the best method to use in teaching children a particular concept or idea. It has been proven through past studies that children learn in different ways. Another way of stating this is that there are various learning styles related to children. Some children may learn visually, while others may learn better auditorily. The tactile mode may also be another way children learn. There are also those who use a combination of modalities mentioned in order to understand or comprehend an idea. The visual learners learn by actually seeing the illustration of what they are to learn, whether it is represented through a picture, a diagram, or even a model. The learners that learn auditorily use the sense of hearing to grasp an idea or concept. This may be accomplished through the use of such devices as tape recorders, record players, or even simple conversation. The tactile mode involves the sense of touch. With this mode, the student has to actually experience by touching the actual object being discussed.

In view of the modalities, which are ways in which one can learn, there are certain levels of learning or intellectual development. Various educators, along with psych-

ologists, have formulated various levels of learning that they feel a student goes through in order for the concept to be understood. Bruner, for example, has devised three levels of learning that a student goes through.[1] These levels are the enactive, the iconic, and the symbolic. The enactive level, most often refers to the level of concreteness. This level is concerned with concrete objects. The student is presented with the actual object, in order that he may manipulate and look at it to formulate various ideas about the subject matter. After the inactive level, there is the iconic level. This level refers to the pictorial aspect of representation. At this level, picture of objects, diagrams, or sketches are used in helping students to understand a particular concept. The third level is the symbolic level. This level uses symbolism such as words or numbers to capture an idea. At this level, the student no longer needs the concrete presentation or pictorial presentation of a concept, but can understand by just using the symbols. An example of this level would be as follows: when a child is presented with "2+3" he knows what 2 and 3

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[1]Jerome S. Bruner, Toward a Theory of Instruction (Cambridge: Harvard University Press, Belknap Press, 1963) p. 28 cited by Linda Barron, Mathematics Experiences for the Early Childhood Years, (Columbus: Charles E. Merrill Publishing Co., 1979), p. 4.

represent so that he will be able to find the answer which is 5. At the enactive level he would have to use two blocks plus three blocks to get his answer Then, at the iconic level, he would need a picture of three marbles plus two marbles in order to grasp the concept of addition. An activity may include all three of these levels, or two, or even just one. The level used is determined by the needs of the learner. Some students may need the concrete representation, while others only need the pictorial representation.

Gagne', another psychologist, has developed a hierarchy of learning.[1] A student progresses from one stage to the other in the hierarchy. The first stage is known as signal learning. This stage involves a generalized emotional response. The response is essentially involuntary. Some of these responses include crying, sucking, and smiling. The next stage is called stimulus-response learning. In this stage the stimulus that causes a response is singled out. When this occurs there is a reward for the correct response, or a punishment for the incorrect response. The third stage is chaining. All that chaining consists of is

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[1]Robert Gagne', The Conditions of Learning, 2nd ed. (New York: Holt, Rinehart and Winston, Inc., 1970), cited by Linda Barron Mathematics Experiences for the Early Childhood Years, (Columbus: Charles E. Merrill Publishing Co., 1979) p. 6-8.

the sequencing of two or more stimulus response situations. The fourth stage is verbal associations. Verbal associations is characterized by a verbal sequence. No longer do we have just motor activities but a verbal representation comes into focus. An example of this is shown through memorization of the basic facts of addition. The verbal association occurs when the student has to say the basic facts of addition from memory. After verbal association, discrimination is next. At this stage, a student should be able to respond correctly to each of several stimuli that are given. In terms of mathematics, the student would have to be able to name the numbers correctly when the numerals are given in random order. The next stage is called concept learning. This is the sixth stage. The student at this stage should be able to place objects in the environment into the classes they belong to. This skill involves recognizing something that is common to those objects, such as size or shape, and thereby grouping them together. Rule learning follows concept learning. The student formulates rules based on the relationship between two or more concepts. An example of rule learning would be the ability to know which sums are even, and which are odd, when adding even numbers or odd numbers. The problem solving level is the last one according to Gagne. This level refers to rules which form higher level principles.

The student is able to solve problems within his environment. An example of this level would be a situation whereby the student is in the store and has to buy apples and oranges for a group of people. Five persons want oranges, and five want apples. The student has to be able to relate this situation and know that he has to buy five apples, and five oranges, and have a total of ten fruits.

The learning hierarchy is not necessarily equated with a particular age range. A student progresses from one stage to the other after being able to learn at that stage. Jean Piaget, on the other hand, has identified four levels of intellectual development.[1] These stages are related to age ranges. The first stage is known as the sensory-motor stage. This stage begins at birth and ends at about two years of age. The child at this stage engages in reflex actions such as crying, sucking, and grasping. These actions are ways of experiencing the environment. After the sensory motor stage, the preoperational stage begins. It begins around the age of two and continues to the age of seven. At this stage, the child is not able to reason from someone else's point of view, only his own point of view. He is unable to perform conservation tasks

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[1]Jean Piaget and Barbel Inhelder, The Psychology of the Child, (New York: Basic Books, Inc., 1969), cited by Linda Barron, Mathematics Experiences for the Early Childhood Years (Columbus: Charles E. Merrill Publishing Co., 1979), p 2-3.

such as seeing that the quantity of a set of marbles remains the same regardless of the sizes of the containers they are placed in. For example, if six marbles were placed in a large container, and six were placed in a smaller container, the child has a tendency to claim that one container has more than the other when in reality they have the same amount. The concrete operational stage begins at about seven and ends at about eleven years of age. This stage is where we see the emergence of logical thought. The student forms concepts based on his contact with concrete objects. This is especially evident in the area of mathematics where the student learns to count by having the concrete objects in front of him. He no longer is unable to see that the six marbles he starts out with are what he ends up with regardless of the size of the container they are placed in. The formal operational stage is the last stage of Piaget's developmental scheme. This stage begins approximately at age eleven or twelve and continues to age fifteen. The student is no longer dependent on concrete objects to formulate ideas. He can now formulate predictions, reason deductively, and understand hypothetical situations. The stages of learning are very important to the educator in terms of finding ways in which to educate the youngsters.

Characteristics of Educable Mentally Retarded Students

Before looking at the characteristics of the educable mentally retarded student, a definition as to who are they is needed. The American Association of Mental Deficiency defines it as the following: "Mental retardation refers to significantly subaverage general intellectual functioning existing concurrently with deficits in adaptive behavior and manifested during the developmental period."[1]

Mental retardation is divided into categories. These include educable mentally retarded, trainable mentally retarded, and the severely mentally retarded. The educable mentally retarded may have difficulty in the area of learning abstract concepts. Their appearance does not look different from that of normal children. Educable mentally retarded is categorized by an intelligence quotient range of 50 to 70 or 75. There are certain characteristics to be found in the educable mentally retarded student. One characteristic is that he is sensitive to his surroundings; this child seems to know when he is accepted or not. Researchers have advised that the child be shown alot of love and praise, instead of rejection. The important point here is the value of acceptance. According to Malinda Garton, "Acceptance is

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[1]William R. Van Osdol and Don G. Shane, An Introduction to Exceptional Children, (Iowa: Wm C. Brown Co., 1974) p. 57.

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important for the preservation of the child's dignity and the achievement of self realization."[1] This acceptance goes a long way in terms of the child's achievement. He would try his best to do well, knowing that someone cares about him. Another characteristic of this child is that he has a slow reaction time. He is unable to become interested in a new activity without some adjustment. There should be time for him to get adjusted to a new activity. Some time should be alloted for the student to put away materials of the previous activity before engaging in a new one. His attention span is short. With this in mind, the child has to be actively involved in the learning activity. The materials being used in the activity should be at the child's level of interest and comprehension. This is very important in enabling the child to complete the task with a feeling of satisfaction. In terms of language, the educable mentally retarded child has difficulties in the "use and comprehension of verbal and numerical symbols."[2] This child may not understand what is meant by time, or even the value of planning. He has a difficult time understanding the value of money. Along with this, he seems to be of the practical type. What

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[1]Malinda Garton, Teaching the Educable Mentally Retarded, (Springfield: Charles C. Thomas, 1964) p. 18.

[2]Ibid., p. 21.

is meant by this is that it seems as though he is unable to apply separate qualities to the solution of problems.[1] The educable mentally retarded student has little ability in terms of self-evaluation of his efforts. He has a narrow range of interests. However, according to Malinda Garton, this range of interests can be stimulated by use of audiovisual materials, field trips, and dramatizations. Other difficulties of these students include difficulty in recognizing boundaries, difficulty in distinguishing right and wrong, and difficulty in terms of emotional stability. He, however, has the ability to be loyal and acquire habits. The above mentioned characteristics are those that are often seen with educable mentally retarded students.

Characteristics of Educable Mentally Retarded Students in Arithmetic

According to Burns, the educable mentally retarded student is retarded in the knowledge of arithmetic vocabulary, ability to solve abstract and verbal problems, understanding the concept of time and sequence, and differentiating extraneous material from needed arithmetical facts.[2] These

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[1]Ibid., p. 22.

[2]Paul C. Burns, "Arithmetic Fundamentals for the Educable Mentally Retarded," American Journal of Mental Deficiency, 66 (1961) p. 58.

students are more careless at their work, make more technical errors, and use more immature processes than their normal peers. Arithmetic readiness is very important for these students. They do better in addition and subtraction than in multiplication and division. Not only this, but they do well in word problems when the situations of the problems are meaningful to them.

Another set of behaviors in regard to arithmetic was stated by Garton in her book entitled, Teaching the Educable Mentally Retarded. This author claims that the following behaviors are characteristic of the educable mentally retarded. These include: low transfer of learning, low abstract thinking ability, poor observation and comprehension of details and situations, slow absorption of facts, little initiative, and lack of ability to concentrate.[1] With these characteristics in mind, these students must be taught in such a way that they can understand, and this understanding can in turn be related to real life experiences.

Other research has indicated that these students perform up to mental age expectancy on computational skills, and functional areas such as time and money.[2] They do poorly on situations that involve concept development and

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[1]Garton, Teaching the Educable Mentally Retarded, p. 220.

[2]Frank Hewett, Education of Exceptional Learners (Boston, Allyn and Bacon, Inc. 1974), p. 367.

reasoning. These students may engage more often in elementary means of obtaining an answer, like counting of fingers, than their normal peers. Unlike their normal peers, these students do only half as much academically as their normal peers. Therefore, it is essential that what they do learn is beneficial to their everyday life.

Arithmetic That Is Needed

Based on the characteristics of the educable mentally retarded student, his arithmetic should be taught in small step sequences which are designed to produce success. This student has to obtain a level of success at his tasks or he becomes easily frustrated. Another type of activity is suggested for this student; this activity is called individualized instruction. He is given instruction based on his particular level of achievement and knowledge. He doesn't have to keep up the pace with his fellow students but he achieves based on his own ability and interest. Other activities include small group involvement. The student should also be able to work with concrete materials. It is suggested by researchers that physical activity be a part of these students' instructional experiences.[1] By physical activities,

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[1]Kenneth Lovell, The Growth of Basic Mathematical and Scientific Concepts in Children (London: University of London Press, 1961), Leo Brueckner, Foster Grossnickle and John Reckzeh, Developing Mathematical Understanding in the Upper Grades, (Philadelphia: J. G. Winston, Co., 1957), Zoltan P. Dienes, Building Up Mathematics, (London: Hutchinson Educational, 1960) cited by Austin Connolly, "Research in Mathematics Education and The Mentally Retarded", Arithmetic Teacher 20, (Oct. 1973) p. 495.

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researchers refer to activities where the students actually move things with their hands such as concrete materials. The use of concrete action-oriented teaching aids by students is supported by Piaget, Bruechner, Grossnickle, Reckzeh, and Dienes.[1] In the arithmetic activities, reading should be kept at a minimum. Realizing the type of student one is working with, the standards of evaluation should be reasonable, based on the capabilities of the student. The diagnostic and evaluative techniques should be used frequently. This should be constantly going on so that progress may be noted and also areas of remediation. The diagnostic and evaluative techniques enable educators to make judgements as to what the student knows, doesn't know, and needs more work on.

Another suggestion is brought about concerning the arithmetic of the students. It has been suggested by Burns that these students be taught arithmetic methodically.[2] They need frequent review and maintenance exercises. The concepts that have to be taught will have to re-introduced almost every year. The review and re-introduction is needed because concepts have to be reinforced through these methods or they have a tendency to be forgotten. The levels of

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[1] Ibid., p. 495

[2] Burns, "Arithmetic Fundamentals for the Educable Mentally Retarded," p. 57.

instruction should, as much as possible, refer to specific experiences and situations to which the student can relate, and therefore get a better understanding of the concepts being taught.

There are three major jobs for the teacher of the educable mentally retarded when it comes to arithmetic. The teacher has to decide, "what is the most important to teach these students, understand the weaknesses of these students in the area of arithmetic, and decide upon methods and materials for instruction."[1] Burns, in his article feels that, based on research, it would appear that the major emphases to be placed in arithmetic are the following:

> "a strong arithmetic readiness program, use of concrete materials and manipulative aids, a variety of activities and procedures for each skill, use of grouping and individualized instruction; following an orderly system and sequence; use of considerable oral and incidental arithmetic; meaning and understanding on numbers as contrasted with mere mechanical manipulation of numbers; use of computational skills in meaningful, life-like, socialized situations; and considerable recurrence, distributive rather than concentrated.[2]

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[1]Ibid., p. 59.

[2]Ibid., p. 59.

Johnson and Mykelbust seem to view the disorders in arithmetic that the educable mentally retarded student may have as stemming from two basic problems. These problems are those in other language areas and disturbances in quantitative thinking.[1] If a student is having problems in the language area such as auditory receptive language, he may have problems in arithmetic. This may be the result of him not being able to profit from the teacher's verbal presentation of the principles. In this area, the student may not be able to understand word problem contexts which are utilized in the spoken instruction that is given by the teacher. If the student has difficulty in reading, he may have some trouble interpreting word problems. He may have difficulty in writing down his answers correctly due to poor visual motor integration.

The other aspect that was mentioned before was disturbances in quantitative thinking. Disturbances in this area may result in the student having problems in comprehending certain mathematical principles. In order for the student to acquire the skill in understanding and using quantitative relations, instruction must begin at the basic, non-verbal

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[1]D. L. Johnson and H. Myklebust, Learning Disabilities: Educational Priciples and Practices, (New York: Grune and Stratton, 1967) cited by Frank M. Hewett, Education of Exceptional Learners, (Boston: Allyn and Bacon, Inc., 1974) p. 367.

level. The principles of quantity, order, size, space, and distance must be taught. Connolly found that the developmental sequence proposed by Piaget, that was presented earlier in this study, is relevant to mentally retarded individuals also. The Piaget-type tasks, such as the conservation of quantity, are a function of the mental age of these children. According to Piaget, the notion of numbers, along with other mathematical concepts, are not learned just from teaching.[1] He feels that to a large degree, these ideas are developed by the child himself. The true understanding of the concepts involved comes about with the mental growth of the child. Piaget also feels that, if a concept is introduced prematurely, the learning that takes place is only verbal; true understanding occurs as the child grows mentally and is at the mental age to understand the concept. A child needs to understand the concept. A child needs to understand the principle of conservation of quantity before being able to develop the concept of numbers.

The work of Piaget has certain implications for the teacher engaged in the task of teaching elementary school mathematics. Some implications are as follows:

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[1]Jean Piaget, "How Children Form Mathematical Concepts". Scientific American 189 (Nov. 1953) p. 74.

1. The child's mental growth advances through qualitatively distinct stages. These stages should be looked upon when planning the curriculum.

2. Test the student to be sure he has mastered the prerequisite for that concept before introducing a new concept.

3. The pre-adolescent child makes typical errors of thinking based on his stage of mental growth. The teacher should try to understand these errors.

4. The teacher can help the child to overcome errors in his thinking by providing experiences to show the errors and ways to correct the errors.

5. The student in the pre-operational stage has a tendency to fix his attention on one variable and neglect the others. The teacher should help him overcome this by providing many situations so he may explore the influence of two or more variables.

6. Teachers should teach pairs of inverse operations in arithmetic together because a child's thinking is more flexible when it is based on the reversible operations.

7. Mental growth is encourged by experience of seeing things from many different points of view.

8. Physical action is one of the bases of learning. To learn effectively, a child must be a participant in the events.

9. There is a lag between perception and formation of a mental image. We can reinforce the developing mental image with frequent use of perceptual data.[1]

Cruickshank, in his research on mentally retarded youngsters, found results that were similar to those that were mentioned before. He found that the mentally retarded youngster was inferior to mental age normal peers in:

1. Ability to solve abstract and verbal problems.
2. Ability to solve concrete problems.
3. Their understanding of the operations to solve a problem.
4. Their ability to isolate pertinent information from a body of given data.
5. Their work habits which are characterized by carelessness and immaturity.[2]

Bower did a study on mentally retarded and normal children in which a comparison of their arithmetic competencies was made. A field test version of Key Math was used. He

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[1]Vincent Glennon and Leroy Callahan, <u>Elementary School Mathematics</u>: A Guide to Current Research, 3d ed. (Wash. D.C.: Association for Supervision and Curriculum Development, NEA, 1968) p. 16.

[2]W. M. Cruickshank. "A Comparative Study of Psychological Factors Involved in the Response of Mentally Retarded Children Ages Thirteen through Sixteen (Ph.D. dissertation, University of Michigan, 1946) cited by Austin Connolly, "Research in Mathematics Education and Mentally Retarded" <u>Arithmetic Teacher</u> 20 (Oct. 1973) p. 492.

found that the performance of the mentally retarded was inferior to that of the normal students with the same chronological age. However, when the mentally retarded were compared to normal students of similar mental age they were superior in certain areas. These areas were multiplication, division, money, time, and calendar. The normal youngsters were superior to the mentally retarded in addition, subtraction, numerical reasoning, and measurement. The results of this particular study implied that mentally retarded students perform best on computational and functional areas of arithmetic, weaknesses in areas of arithmetic requiring verbal mediation and weaknesses in work habits typified by careless computational errors, following directions, and organizing their work.[1]

There were three instructional practices or approaches suggested to be used in the arithmetic instruction for the mentally retarded.[2] According to Connolly, one approach stresses language and verbal information processing. Another is known as the manipulative and discovery approach. The third approach is the individualized

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[1]N. Bower. "A Comparison of Arithmetic Competencies by Mentally Retarded and Normal Children" (Master's Thesis, University of Missouri, 1970) cited by Austin Connolly, "Research in Mathematics Education and Mentally Retarded", p. 493.

[2]Austin Connolly. "Research in Mathematics Education and Mentally Retarded," Arithmetic Teacher 20 (Oct. 1973) p. 495.

instruction approach. Bereiter and Engelmann place emphasis on the approach that stresses language.[1] They feel that language is the cornerstone of math instruction. The idea that language is central to academic learning is the premise for this approach. With this approach, emphasis is placed on learning to manipulate and interpret arithmetic statements based on consistent rules. The language of arithmetic is looked at.

The second approach, manipulative and discovery approach, looks at learning through concrete action-centered learning materials. These materials provide the students with experiences so that abstractions and concepts can be understood. The third approach, individualized instructions, may incorporate some of the elements of the other two. This approach is based on frequent diagnostic assessments. The student works on a series of tasks based on his own rate. His performance on the previous task dictates what the next set of tasks will be. Based on the three approaches presented, Connolly suggests a teaching arrangement where the approaches are combined. This arrangement is as follows:

Step 1. The concepts are introduced. Explanation of the subject matter has to be in general terms. The activity is related to the child's past experience

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[1]C. Bereiter and S. Engelmann. Teaching Disadvantaged Children in the Preschool (Englewood Cliffs, N.J.: Prentice-Hall, 1966) cited by Austin Connolly, "Research in Mathematics Education and Mentally Retarded:, p. 495.

and skills.

Step 2. Manipulative materials are provided to be used in groups and by individuals.

Step 3. The child has to orally summarize the concept he has learned.

Step 4. Individual materials are provided for the child. These materials help to verify that the child has understood the concept and can apply it.[1]

Manipulative Devices

Under the umbrella of manipulative devices, we find the Cuisenaire rods. Manipulative devices are those concrete materials, which when handled by the student, enables him to attain the objective or objectives that have been identified. Since Cuisenaire rods are manipulative devices, it is appropriate that there be a focus on manipulative devices presented here. It has been noted that manipulative devices are essential to the instruction program for arithmetic. According to Van Engen, "reactions to the world of concrete objects are the foundations from which the structure of abstract ideas arises."[2]

According to certain researchers, manipulative devices seem to be good devices to use in the learning of

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[1]Connolly, "Research in Mathematics Education", p. 496.

[2]Henry Van Engen. "The Formation of Concepts", In The Learning of Mathematics: Its Theory and Practice (Washington D.C.: The National Council of Teachers of Mathematics, 1953) p. 69-98, cited by Emil Berger ed. Instructional Aids in Mathematics (Wash., D.C. Nat'l Council of Teachers of Mathematics, Inc., 1973) p. 302.

arithmetic.[1] In research conducted by Dienes, it was understood that the organism seems to wish to explore and manipulate the environment. By doing this, it is able to predict how the environment is going to respond. Adler, a psychologist, feels that physical action is one of the basics of learning. Here again, we see the need for the child to actively participate in the learning activity in order for the learning to occur. Gagné feels that instruction needs to be fundamentally based on the stimulation that is provided by objects and events. He also feels that a child can learn better, especially arithmetic, when he has an abundance of opportunities to manipulate physical objects. These objects are the stimuli, through which concepts are formed. Such learning theories as Piaget's, Bruner, and Flavell's agree that the performance of internal operation accurately is increased by the experience one gets with concrete materials.

Much of the research concerned with manipulative material is inconclusive. However, there are those who show positive results toward the use of manipulative devices. Harvin, for example, used questionnaires in his study and found that the frequency of the use of such materials appears to be a contributing factor in achievement in mathematics. Adkins and Suddeth found in their study that

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[1]Berger, Instructional Aids in Mathematics, p. 302.

their is a tendency to use more instructional materials in primary grades for motivation, influence of attitude, and the purpose of discovering relationships. Sole, on the other hand, found that the use of a variety of materials does not produce better results than a single device if both of these methods were used for the same amount of time. He feels that regardless of what materials and how many materials are used it is the amount of time spent that is important. If more time is spent, achievement is improved.

According to Bernstein, there are certain principles on the selection and use of manipulative materials. They are:

1. There should be correlation between the operations carried out with the device and those carried on in doing the mathematics with pencil and paper.

2. The aid should involve some moving parts so that it illustrates the mathematics principles that are involved.

3. The device should exploit as many senses as possible.

4. The student should have his own device along with ample time to use it.

5. Learning may proceed from using physical models to using pictures to using only symbols.

6. The use of manipulative devices should be permissive rather than mandatory to the child.

7. The device should be flexible and have many uses.[1]

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[1]Allen Bernstein, "Use of Manipulative Devices in Teaching Mathematics," The Arithmetic Teacher, 10 (May 1966) p. 280.

The purpose of manipulative devices is to convey an idea. A student is aided in understanding a concept by using a manipulative device. With this purpose in mind, one has to be careful in choosing the manipulative device. Hamilton suggests some characteristics that may help in the decision. The Characteristics are:

1. The outcomes and organizations of the device must not be obscure.
2. Variety is provided.
3. The device is simple to operate.
4. The device should be easy to assemble and store.
5. The parts should not be easily lost.
6. The device should encourage communication of some sort.
7. The device should not be an end in itself.[1]

Other characteristics that should also be included are durability, attractiveness, simplicity, size, and cost of the device. The device should be able to withstand regular use by children. It should appeal to children and be designed so that it is easily manipulated by them. The cost should not provide a true embodiment of the mathematical concept to be explored along with a basis for abstraction.

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[1] E. W. Hamilton, "Manipulative Devices", The Arithmetic Teacher, 13 (October 1966): 462.

There are certain uses of manipulative devices as proposed by Reys. Manipulative devices are used to vary instructional activities, provide experiences in actual problem-solving situations, provide concrete representations of abstract ideas, provide an opportunity for students to discover relationships and formulate generalizations, provide active participation by the pupils, provide for individual differences found in pupils, and increase motivation.[1] The manipulative device, with all its uses, should be selected wisely. It should not be a substitute for teaching, but a convenient aid in the process of learning. The device should be introduced to the student in such a way that he feels comfortable using it and be able to ask questions about it, along with making errors and even correcting these errors for himself.

Many teachers have accepted the use of manipulative devices and visual aids in teaching of arithmetic. There was a study conducted which looked at the effectiveness of selected materials for teaching arithmetic. This study used three groups of first graders. It took place in Oak Park, a Michigan school, during the full school year of 1960-61. They used three sets of manipulative materials One group used a commercial set of devices called Numberaid.

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[1] R. E. Reys, "Consideration for Teachers Using Manipulative Materials," The Arithmetic Teacher, 18 (Dec. 1971): 555.

The second group used a set that was selected by the teacher, and the third group used an inexpensive set of materials. The children in the study were given achievement tests and attitude surveys. Group one's materials included a Numberaid abacus for each pupil, a demonstration model, workbooks, and guidebooks for parents. The cost per pupil was five dollars. Group two's materials consisted of ten bead factfinders, fifty plastic sticks, ten round cardboard discs, and hand-operated adding machines. The materials were used to supplement the text book. The cost per pupil was one dollar. Group three's materials consisted of text books and materials found in the first grade. These were supplemented by homemade materials of the teachers. The three groups engaged in certain aspects of mathematics such as counting, addition of whole numbers, subtraction, along with some multiplication and division. The study, which involved 654 students, found that there were no significant differences in the groups, based on arithmetic computation, reasoning, and total achievement when the mean score for I. Q. subgroups 125 and above and 99 and below were used. There were no significant differences in attitude. Based on these data, the researchers felt that expenditures for manipulative devices don't seem justified.[1]

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[1]Hardwick W. Harshman, David W. Wells, and J. Payne, "Manipulative Materials and Arithmetic Achievement in Grade I. The Arithmetic Teacher, 9 (April 1962): 191.

Another study that involved manipulative devices was conducted in the Santa Clara County. Three elementary schools were chosen, one second grade class from each school. A series of twenty lessons for thirty-five minute periods were conducted in the three modes. The modes were manipulative (M), pictorial ()), and abstract (A). The instructional period began on April 29, 1972 and ended on April 31, 1972. Their knowledge of the concept was measured by an investigator-prepared instrument. The test measured the students' ability to use multiplication concepts abstractly, the use and interpretation of numerals, operations and relation of symbols, and facility with mathematical sentences. There were no significant differences between the manipulative and pictorial groups in their ability to affect the children's concept formation in beginning multiplication.[1] This study gives no evidence to prove that concrete materials contribute more to concept formation than pictorial materials.

Finely conducted an experiment with fifty-four educable mentally retarded students. The students were presented with twenty problems in the concrete, symbolic, and pictorial modes. The concrete mode used money and

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[1]Lloyd Scott and Herman Neufeld, "Concrete Instruction in the Elementary School Mathematics: Pictorial vs Manipulative," School Science and Mathematics 76 (January 1976): 70.

actual objects where the students were tested individually. Problems involving pictures were administered as a group. the symbolic aspect was administered to the group. It was found that the educable mentally retarded performed best on the symbolic test and worst on the concrete test.[1] The experimenter wondered that the concrete test was given individually there may be a possibility that the student felt more at ease in the group than when he was singled out. Along with this, there was a lack of control of important variables such as the teacher variable, and the effects of practice.

In light of what has been previously mentioned, Smith feels that practical application should be stressed in teaching mathematics to the mentally retarded. He also feels that such instruction as money value should start with the manipulation of real things.[2] Here the need for manipulation of the concrete is emphasized.

Weber studied the effect of reinforcement of mathematics concepts. This was done with first graders. She used paper and pencil follow-up activities or manipulative materials for follow-up activities. A standardized

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[1]Carmen Finely. "Arithmetic Achievement in Menally Retarded Children: The Effects of Presenting the Problem in Different Contexts," American Journal of Mental Deficiency, 67 (Sept. 1962), p. 281-86 cited by Glennon, Elementary School Mathematics, p. 46.

[2]Frank Hewett, Education of Exceptional Learners (Boston: Allyn and Bacon, Inc., 1974) p. 369.

test was given which showed that there were no significant differences in the activities providing reinforcement. However, there is a trend that favored the groups using the manipulative materials. These students scored significantly higher on the oral test of understanding.[1]

Lucas conducted a study with first graders. His study involved attribute blocks. The purpose of the study was to look at the effects of attribute blocks on the first graders. His results were quite positive. They showed that the children illustrated a greater ability to conserve number and conceptualize addition-subtraction relations.[2] Based on what has been mentioned, there seems to be a positive feeling toward manipulative devices and their place in mathematics instruction.

Studies Involving The Cuisenaire Rods

There have been some studies done with the Cuisenaire rods. Some of the studies involved normal children, while others involved the educable mentally retarded

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[1]Audra W. Weber, "Introducing Mathematics to First Grade Children: Manipulative vs Paper and Pencil," Dissertation Abstract International, 30A (Feb. 1970): 3372-73 cited by Joseph Payne, ed., Mathematics Learning in Early Childhood (Virginia: Nat'l Council of Teachers of Mathematics, Inc., 1975), p. 53.

[2]Joseph Payne, ed. Mathematics Learning in Early Childhood, (Virginia: Nat'l Council of Teachers of Mathematics, Inc., 1975) p. 53.

children. Those studies which involved the normal children will be reviewed.

Haynes conducted a study where the effectiveness of the Cuisenaire method was compared with a conventional method of teaching multiplication. Five third grade classes were in the study. The conventional method was that method set forth in the book, The New Discovering Numbers. The Cuisenaire method was based on the contents found in Mathematics With Numbers in Color: Book A and A Teacher's Introduction to the Cuisenaire-Gattegno Method of Teaching Arithmetic. A total of 106 students were in the study. One teacher in each of the two schools taught a control and a Cuisenaire group for six weeks. A third teacher taught using only the conventional method. The students were given several tests, including the Metropolitan Achievement Tests and Primary Mental Abilities Tests. This study showed that the Cuisenaire method was no more effective in teaching multiplication to third graders than the conventional method.

Another study was conducted by Crowder. His study looked at the effectiveness of the Cuisenaire method as compared to the conventional method in teaching arithmetic to first graders. An arithmetic achievement test and an inventory were used to measure the outcome of the study. Three hypotheses were tested: 1) The experimental group's arithmetic achievement was significantly greater than the

control group's achievement, 2) Sex seems immaterial to the ability to learn arithmetic in grades one and three. 3) There would be very little difference in achievement between upper and middle groups classified by socioeconomic status. The results of the study were:

> The pupils using the Cuisenaire method learned more conventional subject matter, more mathematical concepts and skills than those that were taught using the conventional program. The average and above average students profitted most from the Cuisenaire method. Sex did not matter in terms of achievement. Socioeconomic status is an important factor in ability to learn arithmetic in first grade.[1]

It seems as though the use of objective materials helps learning of arithmetic in first grade.

Lucow conducted a study in which he used third graders. There was a Cuisenaire group and a control group in each of the eight schools. A pretest and a posttest were administered. The experimental period lasted six weeks, starting January, 1962. It was noted that the pretest revealed that the Cuisenaire group was ahead of the control group. At the end of the experiment the following conclusions were made:

> Cuisenaire method proved to be effective for teaching third grade mathematics and other methods of instruction are also effective.

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[1]Alex Blecher Crowder, Jr., "A Comparative Study of Two Methods of Teaching Arithmetic in the First Grade," Dissertation Abstracts, (Ph.D. dissertation, No. Texas Univ., 1965)

> There was evidence that the Cuisenaire method operates better in a rural setting than an urban setting.
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> The Cuisenaire method operates better on high I. Q. and middle I. Q. in a rural setting, but not much better with low I. Q.
>
> There is a slight indication that girls take to the Cuisenaire method better than boys.[1]

The experimenter felt that the results that favored the Cuisenaire group could be based on the presence of over-aged pupils, repeaters, and deviants in the control group and the differences in mental set toward multiplication and division (The process new to control group but familiar to the Cuisenaire group). In his summary of the study, Lucow feels that the Cuisenaire method is a good one and it should be added to the third grade teacher's repetoire. He also feels that the children should be taught by whatever method they respond to. With the fact that children have individual differences, it is felt that a teacher should not limit himself to one method of instruction.

Another investigator, Passy, wanted to find out the effects of the Cuisenaire method on reasoning and computation. The program was limited to the first three grades and Kindergarten. 1,200 children at each grade level participated. They were tested in May, 1962 with the Stanford Achievement

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[1]William Lucow, "An Experiment with the Cuisenaire Method in Grade Three," American Educational Research Journal, 1 (May 1964): 166.

Test, Elementary Battery and grade scores on arithmetic reasoning and computation. There were three groups. One used the Cuisenaire method in modified elementary curriculum. The second group used a meaningful arithmetic program (not Cuisenaire). Group three was drawn from the pre-Cuisenaire third grade, in the first group. The first group were third graders. This study revealed that the third graders who used the Cuisenaire method achieved significantly less at the .05 level on the arithmetic subtest of the Stanford and Elementary Battery than the other two groups.[1]

A study was conducted to look at the effects of the Cuisenaire method on the teaching of first grade mathematics. The study began in October 1961 where nine classes of first graders in three schools were selected. Four classes in one school used the Cuisenaire method. Five classes, two in one school and three in another, used the traditional method. Both groups were given a series of three tests at the end of the school year. At the beginning of the 1962-63 school year, the students in the experimental group were assigned to three second grade classes and the controls were assigned to five second grade classes. The instruction lasted twenty-five minutes per day. At the end of the

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[1]Robert Passy, "The Effects of Cuisenaire Materials on Reasoning and Computation," <u>The Arithmetic Teacher</u> 10 (Nov. 1963): 440.

1962-63 school year similar tests were given. Hollis, the researcher, made these conclusions: at the end of the first year and second year, the Cuisenaire method taught traditional subject matter as well as the traditional method and the students in the Cuisenaire group acquired additional concepts and skills to those taught in the traditional programs.[1]

There was a study made in Scotland and England by Brownell. The study was looking at the effectiveness of three programs. These programs were the conventional method (experiences in grouping and use of discrete objects), the Cuisenaire method, and the Dienes method (use of multi-based blocks). The study was concerned with children who had three years of schooling. Schools were selected in Scotland and England (forty-five schools with 1,430 children). There were comparisons made between the Conventional and Cuisenaire groups in the Scottish school. The three programs were looked at in the English schools. The study revealed that the Cuisenaire method was more effective than the Conventional method in developing meaningful mathematical

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[1]Loye Hollis, "Cuisenaire-Gattegno Method with a Traditional Method, A Study to Compare the Effects of Teaching First and Second Grade Math," School Science and Mathematics 65 (Nov. 1965): 685.

abstractions. However, in the English study, the Conventional method ranked the highest in overall ranking for effectiveness in promoting conceptual maturity. The Dienes and Cuisenaire methods ranked equal in this aspect. In this study, there were uncontrolled variables. These were the differences in quality and amount of instruction, the pace of the instruction, and the objectives involved. Although the results seem to point to the Cuisenaire as being more effective, Brownell raised the question that the differences in the results may be due to the skill and enthusiasm portrayed by the teachers and not in the material itself.[1] It may be that the teacher was enthusiastic about teaching something new and this raises the problem of quality of teaching. This variable is very difficult to account for. It seems as though in the English schools, where the novelty of the Cuisenaire method no longer intact, the Conventional method seems to have been better.

An experiment was conducted in Vancouver in the Fall

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[1]Callahan, <u>Elementary Mathematics</u>, 4th ed., p. 15.

of 1957. It consisted of one experimental and one control group in five schools in Vancouver. Four sets of Cuisenaire rods were supplied to the experimental groups. Instruction began in October. The time period of the instruction was twenty minutes a day (ten minutes for teaching and ten for seatwork). They were given four types of tests: A Detroit beginning first grade intelligence test (September, 1957), an initial survey test in number work (January, 1958), a terminal test based on the course of the numberwork for first grade (June, 1958), and survey test for content taught with Cuisenaire materials (June, 1958). The study was conducted during the 1957-58 school year with first graders. The mean score of the Detroit test was higher for the experimental group, but it was not statistically significant. There were significant difference noted in the mean test scores on the survey test of the content taught with the Cuisenaire materials. The performance of the experimental group (Cuisenaire) was superior. This shows that the Cuisenaire method may be more effective than traditional ones with bright and slow children. The conclusions of the study are as follows:

1. There was no significant difference between the experimental and control groups in rate of learning in Grade 1 numberwork.

2. The experimental group surpassed the control group

in terms of the facility with more complex combinations of whole numbers and common fractions.

3. There were no significant differences between the groups on their performance on two problem items that required reading and reasoning.

4. In terms of gains in scores on tests of basic Grade 1 numberwork, the effectiveness of the method of instruction is independent of whether the group is bright or slow.[1]

The following year a second experiment was conducted in Vancouver. The results of this experiment were similar to the one conducted before. An analysis of variance revealed that there was a highly significant relationship between ability and achievement of the groups. There was also a difference in achievement of the groups but it was not significant. Again, the effectiveness of the method was shown to be independent of whether the group is slow or bright.

An experiment was conducted in Saskatchewan during the 1958-59 school year. It was conducted for ten months with 461 in the experimental group and 263 in the control group. A series of tests were administered to the students

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[1]Ivy Hinchliffe. "A Report on an Experiment to Evaluate the Effectiveness of Two Different Methods of Teaching Arithmetic at the Grade One Level," (Thesis, University of Manitoba, 1961) p. 46.

involved in the study. The tests were: power tests administered three times, a special test which had more problems with fractions, and the Pintner Ability Test. After the first year, it was continued for the 1959-60 school year. The experiment involved first, second, and third graders. Based on the results which showed that the Cuisenaire group got higher scores on the tests, it seems as though the method is an effective one. The study was conducted by the Saskatchewan Teachers Federation. A questionnaire was administered to the teachers involved in the study. This subjective data leaned positively toward the Cuisenaire rods. It seems as though the teachers and the students enjoyed working with the rods. After this experiment, another experiment was suggested to be conducted the following year. In 1962 the Saskatchewan Teachers Federation put together a report on the Cuisenaire method. The report consists of principles and procedures of the method. Suggested activities to be used in grades one, two, and three are also in the report. It seems as though the Saskatchewan Federation of Teachers felt that the Cuisenaire method is a good one for teaching mathematics.[1]

Karatzina and Reinshaw conducted a study in Edinburgh during the 1957-58 school year. The experimental

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[1]Saskatchewan Teachers Federation. "Cuisenaire: A Sound Approach to Teaching Mathematics," (Saskatchewan Teachers Federation, 1962) p. 3.

period was eighteen months in which there were forty boys in the group, and fourteen girls and twenty-four boys in the control group. There were three tests administered. They were the Thurstone Primary Mental Abilities Test, Moray House Picture Test, and the Schonell Diagnostic Arithmetic Test. The mean scores of the tests were close, however, there were no statistically significant differences found at the .05 level.

Hinchliffe conducted a study which involved the Cuisenaire method. The study was conducted during the 1959-60 school year in Manitoba. It lasted for ten months. There were 230 first graders in the experimental group and 189 in the control group. This study, like the others, was based on the results of tests that were administered. The experimental group was taught using the Cuisenaire method and the control group was taught using the Living Arithmetic Series. Along with the objective data, subjective data were collected through questionnaires about the advantages and disadvantages of the Cuisenaire method. The questionnaires were given to the teachers and administrators of the classes that were involved in the study. The results of the study are as follows:

1. There were greater gains in achievement of the experimental group.

2. The experimental group exhibited greater skills in

computation.

3. The measurement of ability was in favor of the experimental group.

4. The questionnaires revealed that there was a positive attitude towards the rods.

5. The questionnaires showed that the skills were developed more quickly and easily.

6. The teachers and students enjoyed using the Cuisenaire method.

7. The Cuisenaire method proved to be a challenge for the brighter students.

8. The teachers suggested using the method more.[1]

Another study was conducted using the Cuisenaire method. This study, unlike the others previously mentioned, involved preschoolers. To be exact, the study used five students, each with an average of three years. The rods were used for three months in an attempt to teach them mathematical concepts through free play. There were four girls and one boy, all of average intelligence. The method used by the researcher, Karen Beard, was one in which the children played with the rods only while they were interested in the rods. The knowledge came about through

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[1]Hinchliffe, "Report on an Experiment to Evaluate the Effectiveness of Two Different Methods of Teaching Arithmetic at the Grade One Level," p. 138.

dialogue concerning the rods. These materials hastened the conceptual communication between the children themselves and between the teacher and the children. The researcher made suggestions and invented games in which the students engaged in. They were tested before and after the treatment period. The tests that were used included an awareness test (made up by the researcher), and the Incomplete Man Intelligence Test. The awareness test contained questions on geometry, naming of colors, counting, reading numbers, and relationships among colored lengths of paper.

The students went through four stages while working with the Cuisenaire rods. These stages were: free play (manipulating the rods any way the students wants to explore the rods), free play with directed activities in which the relationships were observed, free play with mathematical notation, and free play with number introduction and written work.[1] Free play is needed so that an atmosphere of creativity and curiosity is established. The purpose of free play are:

1. To present a situation from which a student can unconsciously learn math.

2. To discover what can be done with the rods and see

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[1]Dagny Karen Beard, "An Intensive Study of the Development of Mathematical Concepts Through the Cuisenaire Method in Three Year Olds," (Thesis, Southern Connecticut State College, 1964) p. 31.

what pattern can be made.

3. To provide the teacher with an opportunity to see during the play, what concepts may be forming in the student's mind based on what he does with the rods.

4. For aesthetic pleasure and enjoyment.[1]

In the three months where sessions were conducted three times a week for a period of forty-five minutes to an hour there were some positive conclusions made. Beard feels that the rods were constructive in helping the children to visualize the concept of number, and such operations as addition and subtraction. It was observed that when the student became familiar with the processes involved, he no longer needed the rods and dropped them voluntarily. The students learned by themselves through the dialogue with the rods, observations of others, the involvement in games and discussion about the rods. The results of the study proved the contention of Cuisenaire developers that algebra precedes arithmetic.[2] In working with the rods, the students look at relationships of the rods (for example, one red rod is twice as long as the white rod). This can be written as W+W=R. Algebra is illustrated here by the use of letters. Later, arithmetic comes into play with the

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[1]Ibid., p. 31.

[2]Ibid., p. 55.

use of letters. Later, arithmetic comes into play with the use of numbers.

The studies concerning the Cuisenaire rods that were mentioned before involved the normal children found in the regular classroom. Callahan also conducted a study. He used retarded children in his study. The study started in February, 1966 and lasted for nine weeks. It involved a class of mentally retarded students (nine) whose age range was between seven and ten, and whose I.Q. score range was less than eighty and as low as fifty-seven. The first few classes were devoted to free play of the rods and getting acquainted with the physical properties of the rods. Then they engaged in identifying colors and assigning letter names to the colors. They were taught to build staircases with the rods and to make equations. This introductory period lasted three weeks. The fourth week was spent on developing the basic understanding of addition, subtraction and multiplication facts from one to five. At the end of the fourth week, a test was adminstered. The next five weeks were spent on establishing math facts of numbers six to ten. At the end of the experimental period, the rods were evaluated based on the structure of the child's ability. There were nine important points that came about by this study. These are:

1. The rods satisfied all areas of the Boston School Document of the course of study for the special classes at

the elementary level. These areas are development of an understanding of number concepts, teach the fundamental processes, application of the four processes to a problem, development of an understanding of measurement, and good work habits.

2. The concepts appear to remain with the student after the rods are removed.

3. The number facts were learned from concrete situations.

4. The rods encouraged the ability to recognize reverse operations (subtraction is the reverse of addition).

5. The commutative property is illustrated in a phyical situation by making trains.

6. The rods provide an opportunity for more advanced math than the coventional method.

7. The results achieved by the rods are better than achievement without the rods.

8. The children do not appear frustrated when manipulating the rods.

9. The rods enabled the students to make discoveries and retain information.[1]

The study seems to have results that are positive in terms

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[1]John Callahan and Ruth Jacobson, "An Experiment with Retarded Children and Cuisenaire Rods," The Arithmetic Teacher, 14 (January 1967) p. 13.

of the effectiveness of the materials. It seems as though these materials work pretty well with the educable mentally retarded.

Suggested Mathematics for the Educable Mentally Retarded

The type of math that was suggested for these students was a comprehensive model which stressed verbal information processing. The model would also stress conceptual development and numerical reasoning. This type of model was suggested by Crawley and Vitello.

When teachers were asked what type of mathematics instruction to be used with EMR students, many of them suggested that these students be given more time in working through their arithmetic assignments in the regular math program. Along with this, the notion of more use of concrete materials is generally accepted by teachers of EMR students.

Kirk and Johnson feel that the context to be taught to these students should be chosen on the basis of the following two principles: 1) content must include the knowledge, skills, and concepts that will be of the most value to these students later in life and 2) methods that are used should be determined by the special disabilities or abilities of these children.[1]

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[1] S. Kirk and O. Johnson. Educating the Retarded Child (Boston: Houghton Mifflin Co. 1951) cited by Callahan Elementary School Mathematics, p. 71

Crawley and Goodman stressed the use of manipulative and pictorial devices in working with educable mentally retarded students. These devices seem to help in the learning situation of these students. There was a demonstration program illustrated by Crawley and Goodman. The results showed an improvement in verbal problem-solving and understanding of the principles involved.

Goodstein, Bessant, Thibodeau, Vitello, and Vlahakos conducted a study which looked at the verb al problem-solving ability of the educable mentally retarded student. They found that the use of pictorial aids helped these students obtain a greater degree of performance in that area.

Others suggest the use of programmed instruction. Some of the benefits that were suggested by the use of programmed instruction were a reduction of time required to obtain a skill and a reduction in negativism and hostility toward the method of instruction. However, there is no clear and consistent advantage of this method over others. As a matter of fact, some researchers such as Smith and Quanhebush found that the students seem to need additional reinforcement besides that given by the machines. Most of the students needed approval by the teacher.

It is a common notion among educators that learning should be meaningful. The teacher of an EMR student has to be able to teach what is essential for the students to know and yet teach it in a way that there are developmental and

practice activities that lead to understanding and application. It has been suggested by Callahan that the teacher of these students pay attention to the principles of development that govern the child's mode of thinking, the principles of cognitive adaptation (conservation, equivalence, and flexibility), and the factors that are related to visual perception.[1]

In light of what has been mentioned concerning what type of mathematics to teach EMR students, it seems as though the majority of what is being taught is the same as what is being taught in the regular class. According to Cawley, "There isn't a single comprehensive arithmetic program that has been developed, tested and validated for use throughout school age range for the mentally handicapped children."[2] He also feels that until such a program is established the educator will not know whether the low achievement is the result of poor instruction, the curriculum, or the limitations of the individual. One of the greatest limitations in terms of searching for a good program for the mentally handicapped (another term for EMR) is the notion that these students are concrete learners. This notion has led to a

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[1]Callahan, Elementary School Mathematics, p. 72

[2]John F. Cawley, "Teaching Arithmetic to Mentally Handicapped Children," Strategies for Teaching Exceptional Children, (Denver: Love Publishing Co., 1972) p. 250.

the present generation, along with future generations, may be able to live comfortably in the world.

CHAPTER III

DESIGN

The study looked at the following hypothesis: There will be a significant difference in the mean of the posttest scores of the students using the Cuisenaire rods as compared to the mean of the posttest scores of those students using the traditional method when adjusted for previous mathematics knowledge and intelligence.

SOURCE

The study involved two primary educable mentally retarded classes. One was at the Ulla Muller Elementary School and the other one was at the Peace Corps Elementary School. Eighteen educable mentally retarded students were involved, ten in the experimental group and eight in the control group. The age of the students ranged from nine to eleven years.

PROCEDURE FOR COLLECTING DATA

The study was quasi-experimental in design. Both the experimental group and the control group were tested prior to the treatment period. At the end of the experimental period, a posttest was administered to both groups. The

test that was used was the Stanford Achievement Test Primary Level II. Only the mathematics computation test of the battery was used. The test items were randomly split in half along with the time for administration of the test. One-half of the test served as the pretest and the other half was the posttest. Test time for both tests was fifteen minutes.

Both the control and experimental groups remained in their self-contained classrooms. The researcher taught both the experimental and control groups during the three week period. Throughout the experimental period of three weeks, the experimental group worked with Cuisenaire rods to solve addition and subtraction problems involving whole numbers. The control group used the Mathematics Around Us textbook for grade two, Publisher - Scott, Foresman and Company, along with workbook exercises and drill exercicises placed on the chalkboard. Both groups were taught each day for a period of three weeks for approximately sixty minutes each day. The mathematics problems consisted of one and two digit addition and subtraction problems.

ANALYSIS OF DATA

The data was analyzed by the use of the analysis of covariance. The I. Q. and pretest scores were used as

the covariates. The posttest scores were the dependent variables and the treatment was the independent variable. There was a comparison made of the pretest and posttest scores of both groups to see if any gains in achievement were made. The T Test was used to analyze this.

CHAPTER IV

THE FINDINGS

The research was conducted over a period of three weeks where two groups of educable mentally retarded students were taught the above mentioned concepts. One group, the experimental group, used the Cuisenaire rods while the other group, the control group, used the traditional method. Both groups were taught by the researcher for approximately one hour, five days a week. Each group was given a pretest at the beginning of the three week period, and a posttest at the end of the period. The posttest scores served as the dependent variable, and the treatment received by each group served as the independent variable. The criterion variable is the posttest, and the control variables are the pretest and the I. Q. scores.

Table I presents descriptive data on pre-and post-test and I. Q. scores of the two groups. It was shown by the analysis of covariance that the null hypothesis cannot be rejected at the .05 level of significance (Table II). There were no significant differences in the means on the posttest scores at the .05 level of significance.

The T Test that was used for both groups showed that the difference between the means of the pre- and post-test

TABLE I

MEANS OF PRETEST AND POSTTEST FOR TWO GROUPS OF EMR STUDENTS IN MATHEMATICS

Method	n	Pretest Means	Posttest Means	I.Q. Means
Traditional	8	51.38	74.25	59.75
Cuisenaire	10	52.90	72.30	64.30
Total	18	104.28	146.55	124.05

TABLE II

ANALYSIS OF COVARIANCE FOR ACHIEVEMENT DIFFERENCES BETWEEN TWO EXPERIMENTAL MATHEMATICS GROUPS CONTROLLING FOR PRIOR MATHEMATICS ACHIEVEMENT AND INTELLIGENCE

Source of Variation	Degrees of Freedon	Sum of Squares	Residuals Mean Square
Between	1	56.6	56.6
Within	14	2210.0	157.86
Total	15	2266.6	

$$F = \frac{\text{mean square (between)}}{\text{mean square (within)}} = \frac{56.6}{157.86} = .359$$

CHAPTER V

DISCUSSION AND RECOMMENDATIONS

This study attempted to show the effectiveness of the Cuisenaire Rods with educable mentally retarded students. It was found that there was no significant difference in the two methods employed. There was achievement in both groups but there was no significant difference of one method over the other. Therefore, the method employed did not make much of a difference in the achievement.

Throughout the three week period, there are certain observations made by the researcher that needs to be noted here. In the review of the literature, certain characteristics of the educable mentally retarded were mentioned. Some of these characteristics were observed by the researcher. These characteristics are: 1) The use of fingers in counting, 2) They made a lot of mistakes, 3) Repetition was needed in order for the concept to be maintained, 4) They have a tendency to forget easily and, 5) They were slow in absorbing facts.

Based on the findings of the study, the following recommendations are submitted:

1. A study similar to this one should be conducted for a longer period of time. A suggested period of two

APPENDIX A

SCORES OF TWO GROUPS
(CUISENAIRE AND TRADITIONAL)
ON
A PRETEST (X_1), A POSTTEST (Y), AND I. Q. (X_2)

SUBJECTS	Y	X_1	X_2
TRADITIONAL			
1	59	41	70
2	65	41	64
3	88	53	49
4	76	41	41
5	100	88	71
6	71	71	72
7	76	47	59
8	59	29	46
TOTAL	594	411	478
CUISENAIRE			
9	82	59	68
10	94	53	55
11	71	47	67
12	65	59	55
13	47	41	54
14	76	82	66
15	65	29	74
16	59	59	69
17	76	41	68
18	88	59	67
TOTAL	723	529	643
GRAND TOTAL	1,317	940	1,121

APPENDIX B

PRETEST

Samples A. 3+1=□ 3 4 5 2 NH O ● O O O	B. □ -2=1 5 4 2 3 NH O O O O O	C. 2 3 3 7 5 4 NH +1 O O O O O O
1. 2 +7 8 9 10 7 NH O O O O O O	2. 9 +6 17 14 16 13 NH O O O O O	3. 8 +6 14 3 1 2 NH O O O O O
4. 15 -9 6 14 5 7 NH O O O O O	5. 7 8 +8 22 23 24 21 NH O O O O O	6. 13 -6 13 10 7 5 NH O O O O O
7. 109 -43 143 66 103 43 NH O O O O O	8. 11-4=□ 6 8 7 15 NH O O O O O	9. □+3=11 9 14 7 8 NH O O O O O
10. □-7=6 7 13 1 11 NH O O O O O	11. □ -9=9 17 3 0 18 NH O O O O O	12. 83 +46 139 21 119 129 NH O O O O O
13. 127 -63 64 144 44 180 NH O O O O O	14. 108 -54 104 17 154 54 NH O O O O O	15. 98+9=□ 105 107 26 917 NH O O O O O

Note: The meaning of the symbols below:
> is greater than
< is less than
= is equal to

Samples A. 5 O 3+1 B. 4 O 2+3 C. 2+4 O 3+3	● > O > O >	O < O < O <	O = O = O =
16. 7+8+6 O 8+5+7	O >	O <	O =
17. (7+6)+8 O 6+(6+8)	O >	O <	O =

APPENDIX C

POST-TEST

Samples A. 3+1=□ 3 4 5 2 NH 0 ● 0 0 0	B. □-2=1 5 4 2 3 NH o o o o o	C. 2 3 ÷1 — 3 7 5 4 NH o o o o o
1. 4 +5 8 9 7 10 NH o o o o o	2. 8 +5 12 13 14 11 NH o o o o o	3. 7 -5 1 3 2 12 NH o o o o o
4. 9 7 +8 21 23 22 25 NH o o o o o	5. 43 +63 116 96 16 NH 106 o o o o o	6. 79 -52 26 23 25 28 NH o o o o o
7. 3+4=□ 6 8 7 9 NH o o o o o	8. 2+□=8 10 5 6 7 NH o o o o o	9. □+5=12 NH 7 17 8 6 o o o o o
10. 10-□=4 5 7 6 4 NH o o o o o	11. 63 +55 19 128 118 108 NH o o o o o	12. 175 +73 248 148 1148 247 NH o o o o o
13. 128 -42 160 86 126 26 NH o o o o o	14. 54+7 = □ 59 16 51 61 NH o o o o o	

Note the meaning of the symbols below:

> is greater than
< is less than
= is equal to

	>	<	=
Samples A. 5 o 3+1 B. 4 o 2 3 C. 2+4 o 3+3	>● >o >o	<o <o <o	=o =o =o
15. 5+4 o 4+5	>o	<o	=o
16. 11-8 o 12-7	>o	<o	=o
17. (5+9) 3 o 5+(4+9)	>o	<o	=o

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COLLEGE OF THE VIRGIN ISLANDS

GRADUATE PROGRAM

May 12, 1981

IT IS HEREBY RECOMMENDED THAT THE THESIS BY Vernice Williams

ENTITLED The Effectiveness of Cuisenaire Rods on the Instruction of Educable Mentally Retarded Students

BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

MASTER OF ARTS IN EDUCATION.

COMMITTEE

THESIS ADVISOR

FOR THE

FINAL EXAMINATION

CHAIRMAN, GRADUATE STUDIES

COLLEGE OF THE VIRGIN ISLANDS

Graduate Program

Thesis Oral Examination Report

The consensus of the oral examination committee is that:

Student Vernice Williams

Title The Effectiveness of Cuisenaire Rods on the Instruction of Educable Mentally Retarded Students

Date May 12, 1981

A. Passed-unqualified ____________________

B. Passed-conditionally, based on corrections to be made and resubmitted for approval to the thesis advisor

X

C. Passed-qualified, corrections to be made and submitted to the entire committee

____

D. Failed ____________________

Thesis Advisor

Second Committee Member

____

Graduate Council Examiner

COLLEGE OF THE VIRGIN ISLANDS

THE EFFECTIVENESS OF THE CUISENAIRE RODS ON THE INSTRUCTION OF EDUCABLE MENTALLY RETARDED STUDENTS

A THESIS SUBMITTED TO
THE GRADUATE STUDIES COUNCIL
IN PARITIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE
OF
MASTER OF ARTS IN EDUCATION

BY

VERNICE WILLIAMS

ST. THOMAS, VIRGIN ISLANDS

May, 1981

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CHAPTER I

BACKGROUND

The role of educators has been for some time to teach students, whom they come in contact with, the necessary skills that they will need in order to function within our society. Among the goals of education, one of the most important goals is the transition of knowledge to these youngsters. The knowledge should not be just mere facts and information. Education also has the goal of instilling a desire to learn and to achieve in those students who will be the future leaders in our society. With this in mind, it is hoped that they not only are able to function within our society but to contribute in a positive way to the further development and improvement of the world in which we live. Therefore, the best methods for teaching certain skills so that they may be retained for further use in society are being constantly searched for and analyzed in terms of their effectiveness in learning.

It is a common belief that every child has the right to an education. The educable mentally retarded child is no

exception. He deserves an education also. He was unable to fit into the system of education as we know it. However, due to the passage of Public Law 94-142, every handicapped child has been given the right to an education. The child is no longer forced to change so that he fits into the school system; the school system must change so that it fits into the characteristics of the child. The school system must provide the necessary environment along with the materials to insure that these students develop to their full potential. If the school system does not comply with the law, it can be held liable.

Educable mentally retarded students are considered as part of the group called handicapped or exceptional children. They need to be educated so that they can function within our society and not be considered inferior. One of the most important areas to acquire enough understanding is in the area of mathematics. A knowledge of mathematics is needed in practically every facet of today's society. Mathematics helps us to know the number of days in a month, the amount of any object we may have at a set time, and the amount of money to spend, invest, or save. Not only is the knowledge of mathematics helpful, it has set purposes.

Berger lists the purposes of mathematics as follows:

ulative devices enable the student to actually operate, hold, and really investigate what is placed before him to conceptualize. According to Piaget, learning proceeds through stages.[1] One must start with the concrete level, then to semi-concrete, then to abstract. Here again, we see the need for concrete or manipulative objects for concept learning.

There has not been much research done on the skills of educable mentally retarded students in the area of mathematics. However, the research that has been done shows that certain characteristics are found in the educable mentally retarded students that are typical of that population in terms of mathematics. Glennon summarizes these characteristics as follows:

1. They are retarded in the area of arithmetic vocabulary.
2. They are inferior to normal students in the ability to solve abstract verbal problems.
3. They are better at solving concrete problems than at solving abstract problems.
4. They have less understanding of the processes to be used in a problem situation and are more apt to guess at processes than normal students.
5. They are more careless than normal students at their work, use more immature processes, and make more technical errors.

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[1]Jean Piaget, "How Children Form Mathematical Concepts", Scientific American 189 (November 1953), p. 74.

6. They are less successful in differentiating extraneous material from needed arithmetical facts than normal children.

7. They do equally well with word problems and mechanical operations if the instruction is meaningful to them.

8. They have little concept of time and sequence.

9. They do better with addition and subtraction and need more emphasis on multiplication and division.

10. Arithmetic readiness is even more important to the educable mentally retarded students than to normal students.[1]

With these characteristics, it has been found that this group of students seem to work up to mental age expectancy in mathematical fundamentals. However, this does not occur in arithmetic reasoning in which there is reading and problem-solving. The educable mentally retarded student does develop quantitative concepts in the same order and stages as normal children do. Although they develop these concepts, they are developed later. The concepts are acquired through teaching and maturation. When students are drilled to perform advanced Piagetian-type quantitative tasks, they may appear to have the skills, but they do not really understand the concepts involved. Understanding of the concepts comes

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[1]Vincent Glennon and Leroy Callahan, Elementary School Mathematics: A Guide to Current Research, 3d ed. (Washington, D.C.: Association for Supervision and Curriculum Development, NEA, 1968), p.45.

at a later time after adequate intellectual growth has taken place.[1] According to a number of researchers, concrete materials for teaching mathematics seems to work well with these students. In a similar fashion, it seems as though they learn a particular concept much better when it is presented in the concrete mode. An example of a concept would be addition. In the concrete mode, the child is given two blocks and then two more blocks and asked how many there are all together. The concrete mode is represented through the blocks. The learning style of the educable mentally retarded student is one that encourages him to actually see, hold, and manipulate the material at hand so that it will help him understand the concept. They learn through concrete representation of the subject matter.

Cuisenaire rods emphasize this learning style. They are colored rods that range from one centimeter to ten centimeters long. The student is able to visualize and manipulate the rods. He is able to internalize the concept and gain a better understanding of mathematics by using the rods. These rods were named after Georges Cuisenaire, who invented them. He was a schoolmaster in Belgium. It was because one of his students had problems in understanding mathematics that he started experimenting with pieces of wood. The pieces

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[1]Lloyd Dunn, ed., Exceptional Children in the Schools, (New York: Holt, Rinehart, and Winston, Inc. 1973), p. 148.

were painted various shades of the primary colors, one in black, and the smallest size was not painted. The students in his classroom experimented with them. They soon gained confidence and did mathematical operations with them. Caleb Gattegno was the person responsible for making the rods popular by promoting them abroad. According to Caleb Gattegno, "Color is a factor that is accessible to the minds of almost all humans. Its shades and contrasts can act as a sign to substitute for the abstract notion that it is proposed to attain."[1] It seems as though color would catch anyone's eyes. After this attention is obtained, maybe some learning can be nurtured. The colors for the Cuisenaire rods are based on the following system:

1. All the multiples of two contain the color red, (2cm- red, 4cm- purple, 8cm- brown)

2. All the multiples of three contain the color blue, (3cm- light blue, 6cm- dark green, 9cm- blue)

3. All the multiples of five contain the color yellow, (5cm- yellow, 10cm- orange)

4. Number one was uncolored; number seven was considered the outcast and was colored black.

The Cuisenaire rods focus on the belief that the student sees relationships with the rods in terms of mathematics. These relationships are formulated by the students

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[1]William Ewbank, "The Use of Color for Teaching Mathematics," Arithmetic Teacher, 26 (September 1978): 53.

based on the activities engaged in with the use of the rods. According to the producers of the Cuisenaire rods, the rods capture the attention of students, and this enables students to focus on the task at hand. The philosophy of Cuisenaire rods is one of active teaching. It focuses on seeing, doing, reckoning, understanding, and verification. Based on this philosophy, the student is engaged in manipulation of a concrete set of object, namely the Cuisenaire rods, to work out mathematical operations.

PROBLEM STATEMENT

The question of how to teach the educable mentally retarded has been discussed often. Most researchers feel that they learn best when the subject matter is presented in a concrete manner. In the area of mathematics, the educable mentally retarded have a difficult time conceptualizing the ideas involved. They learn at a slower pace then other students. The particular type of material to be investigated with the educable mentally retarded is the Cuisenaire rods. This material is concrete in its presentation and is used in the area of mathematics. The question of how to best teach the educable mentally retarded mathematics needs to be addressed. How effective are the Cuisenaire rods in helping educable mentally retarded students grasp the concepts of addition and subtraction of one-and two-digit whole numbers? The study will, therefore, look at the effectiveness of Cuisenaire rods on educable mentally retarded students in the area of addition and subtraction of whole numbers (one- and two-digit numbers).

HYPOTHESIS

There will be a significant difference in the achievement of the educable mentally retarded students using the Cuisenaire rods as compared to those using the traditional method.

DEFINITION OF TERMS

1. Educable Mentally Retarded Students A definition as stated by Van Osdol and Shane in An Introduction to Exceptional Children:

   > Mental retardation refers to significantly subaverage general intellectual functioning existing concurrently with deficits in adaptive behavior and manifested during the developmental period.[1]

2. Concrete Materials Those materials which one is able to see, feel, and manipulate.

3. Cuisenaire Rods Colored rods which range in size from one centimeter to ten centimeters long. They are used to improve computational skills and increase mathematical understandings.

4. Traditional Method The use of specified textbook and workbook in teaching mathematics along with the use of drill exercises on the chalkboard with the use of concrete materials when needed.

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[1]William R. Van Osdol and Don G. Shane, An Introduction to Exceptional Children, (Iowa: Wm. C. Brown Company, 1974), p. 57.

LIMITATIONS

The study will be limited to a group of eighteen primary educable mentally retarded students. It will be conducted at two elementary schools on the island of St. Thomas. There will be ten students in the experimental group and eight students in the control group.

SIGNIFICANCE

This study will look at a particular material, the Cuisenaire rods, in terms of its effectiveness with educable mentally retarded students. It is hoped that the study will be of some use to teachers of educable mentally retarded students in their search for appropriate materials for their students.

CHAPTER II

REVIEW OF LITERATURE

In the field of education there is a constant concern about the best method to use in teaching children a particular concept or idea. It has been proven through past studies that children learn in different ways. Another way of stating this is that there are various learning styles related to children. Some children may learn visually, while others may learn better auditorily. The tactile mode may also be another way children learn. There are also those who use a combination of modalities mentioned in order to understand or comprehend an idea. The visual learners learn by actually seeing the illustration of what they are to learn, whether it is represented through a picture, a diagram, or even a model. The learners that learn auditorily use the sense of hearing to grasp an idea or concept. This may be accomplished through the use of such devices as tape recorders, record players, or even simple conversation. The tactile mode involves the sense of touch. With this mode, the student has to actually experience by touching the actual object being discussed.

In view of the modalities, which are ways in which one can learn, there are certain levels of learning or intellectual development. Various educators, along with psych-

ologists, have formulated various levels of learning that they feel a student goes through in order for the concept to be understood. Bruner, for example, has devised three levels of learning that a student goes through.[1] These levels are the enactive, the iconic, and the symbolic. The enactive level, most often refers to the level of concreteness. This level is concerned with concrete objects. The student is presented with the actual object, in order that he may manipulate and look at it to formulate various ideas about the subject matter. After the inactive level, there is the iconic level. This level refers to the pictorial aspect of representation. At this level, picture of objects, diagrams, or sketches are used in helping students to understand a particular concept. The third level is the symbolic level. This level uses symbolism such as words or numbers to capture an idea. At this level, the student no longer needs the concrete presentation or pictorial presentation of a concept, but can understand by just using the symbols. An example of this level would be as follows: when a child is presented with "2+3" he knows what 2 and 3

[1]Jerome S. Bruner, Toward a Theory of Instruction (Cambridge: Harvard University Press, Belknap Press, 1963) p. 28 cited by Linda Barron, Mathematics Experiences for the Early Childhood Years, (Columbus: Charles E. Merrill Publishing Co., 1979), p. 4.

represent so that he will be able to find the answer which is 5. At the enactive level he would have to use two blocks plus three blocks to get his answer Then, at the iconic level, he would need a picture of three marbles plus two marbles in order to grasp the concept of addition. An activity may include all three of these levels, or two, or even just one. The level used is determined by the needs of the learner. Some students may need the concrete representation, while others only need the pictorial representation.

Gagne', another psychologist, has developed a hierarchy of learning.[1] A student progresses from one stage to the other in the hierarchy. The first stage is known as signal learning. This stage involves a generalized emotional response. The response is essentially involuntary. Some of these responses include crying, sucking, and smiling. The next stage is called stimulus-response learning. In this stage the stimulus that causes a response is singled out. When this occurs there is a reward for the correct response, or a punishment for the incorrect response. The third stage is chaining. All that chaining consists of is

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[1]Robert Gagne', The Conditions of Learning, 2nd ed. (New York: Holt, Rinehart and Winston, Inc., 1970), cited by Linda Barron Mathematics Experiences for the Early Childhood Years, (Columbus: Charles E. Merrill Publishing Co., 1979) p. 6-8.

the sequencing of two or more stimulus response situations. The fourth stage is verbal associations. Verbal associations is characterized by a verbal sequence. No longer do we have just motor activities but a verbal representation comes into focus. An example of this is shown through memorization of the basic facts of addition. The verbal association occurs when the student has to say the basic facts of addition from memory. After verbal association, discrimination is next. At this stage, a student should be able to respond correctly to each of several stimuli that are given. In terms of mathematics, the student would have to be able to name the numbers correctly when the numerals are given in random order. The next stage is called concept learning. This is the sixth stage. The student at this stage should be able to place objects in the environment into the classes they belong to. This skill involves recognizing something that is common to those objects, such as size or shape, and thereby grouping them together. Rule learning follows concept learning. The student formulates rules based on the relationship between two or more concepts. An example of rule learning would be the ability to know which sums are even, and which are odd, when adding even numbers or odd numbers. The problem solving level is the last one according to Gagne. This level refers to rules which form higher level principles.

The student is able to solve problems within his environment. An example of this level would be a situation whereby the student is in the store and has to buy apples and oranges for a group of people. Five persons want oranges, and five want apples. The student has to be able to relate this situation and know that he has to buy five apples, and five oranges, and have a total of ten fruits.

The learning hierarchy is not necessarily equated with a particular age range. A student progresses from one stage to the other after being able to learn at that stage. Jean Piaget, on the other hand, has identified four levels of intellectual development.[1] These stages are related to age ranges. The first stage is known as the sensory-motor stage. This stage begins at birth and ends at about two years of age. The child at this stage engages in reflex actions such as crying, sucking, and grasping. These actions are ways of experiencing the environment. After the sensory motor stage, the preoperational stage begins. It begins around the age of two and continues to the age of seven. At this stage, the child is not able to reason from someone else's point of view, only his own point of view. He is unable to perform conservation tasks

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[1]Jean Piaget and Barbel Inhelder, The Psychology of the Child, (New York: Basic Books, Inc., 1969), cited by Linda Barron, Mathematics Experiences for the Early Childhood Years (Columbus: Charles E. Merrill Publishing Co., 1979), p 2-3.

such as seeing that the quantity of a set of marbles remains the same regardless of the sizes of the containers they are placed in. For example, if six marbles were placed in a large container, and six were placed in a smaller container, the child has a tendency to claim that one container has more than the other when in reality they have the same amount. The concrete operational stage begins at about seven and ends at about eleven years of age. This stage is where we see the emergence of logical thought. The student forms concepts based on his contact with concrete objects. This is especially evident in the area of mathematics where the student learns to count by having the concrete objects in front of him. He no longer is unable to see that the six marbles he starts out with are what he ends up with regardless of the size of the container they are placed in. The formal operational stage is the last stage of Piaget's developmental scheme. This stage begins approximately at age eleven or twelve and continues to age fifteen. The student is no longer dependent on concrete objects to formulate ideas. He can now formulate predictions, reason deductively, and understand hypothetical situations. The stages of learning are very important to the educator in terms of finding ways in which to educate the youngsters.

Characteristics of Educable Mentally Retarded Students

Before looking at the characteristics of the educable mentally retarded student, a definition as to who are they is needed. The American Association of Mental Deficiency defines it as the following: "Mental retardation refers to significantly subaverage general intellectual functioning existing concurrently with deficits in adaptive behavior and manifested during the developmental period."[1]

Mental retardation is divided into categories. These include educable mentally retarded, trainable mentally retarded, and the severely mentally retarded. The educable mentally retarded may have difficulty in the area of learning abstract concepts. Their appearance does not look different from that of normal children. Educable mentally retarded is categorized by an intelligence quotient range of 50 to 70 or 75. There are certain characteristics to be found in the educable mentally retarded student. One characteristic is that he is sensitive to his surroundings; this child seems to know when he is accepted or not. Researchers have advised that the child be shown alot of love and praise, instead of rejection. The important point here is the value of acceptance. According to Malinda Garton, "Acceptance is

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[1]William R. Van Osdol and Don G. Shane, An Introduction to Exceptional Children, (Iowa: Wm C. Brown Co., 1974) p. 57.

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important for the preservation of the child's dignity and the achievement of self realization."[1] This acceptance goes a long way in terms of the child's achievement. He would try his best to do well, knowing that someone cares about him. Another characteristic of this child is that he has a slow reaction time. He is unable to become interested in a new activity without some adjustment. There should be time for him to get adjusted to a new activity. Some time should be alloted for the student to put away materials of the previous activity before engaging in a new one. His attention span is short. With this in mind, the child has to be actively involved in the learning activity. The materials being used in the activity should be at the child's level of interest and comprehension. This is very important in enabling the child to complete the task with a feeling of satisfaction. In terms of language, the educable mentally retarded child has difficulties in the "use and comprehension of verbal and numerical symbols."[2] This child may not understand what is meant by time, or even the value of planning. He has a difficult time understanding the value of money. Along with this, he seems to be of the practical type. What

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[1]Malinda Garton, Teaching the Educable Mentally Retarded, (Springfield: Charles C. Thomas, 1964) p. 18.

[2]Ibid., p. 21.

is meant by this is that it seems as though he is unable to apply separate qualities to the solution of problems.[1] The educable mentally retarded student has little ability in terms of self-evaluation of his efforts. He has a narrow range of interests. However, according to Malinda Garton, this range of interests can be stimulated by use of audiovisual materials, field trips, and dramatizations. Other difficulties of these students include difficulty in recognizing boundaries, difficulty in distinguishing right and wrong, and difficulty in terms of emotional stability. He, however, has the ability to be loyal and acquire habits. The above mentioned characteristics are those that are often seen with educable mentally retarded students.

Characteristics of Educable Mentally Retarded Students in Arithmetic

According to Burns, the educable mentally retarded student is retarded in the knowledge of arithmetic vocabulary, ability to solve abstract and verbal problems, understanding the concept of time and sequence, and differentiating extraneous material from needed arithmetical facts.[2] These

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[1]Ibid., p. 22.

[2]Paul C. Burns, "Arithmetic Fundamentals for the Educable Mentally Retarded," American Journal of Mental Deficiency, 66 (1961) p. 58.

students are more careless at their work, make more technical errors, and use more immature processes than their normal peers. Arithmetic readiness is very important for these students. They do better in addition and subtraction than in multiplication and division. Not only this, but they do well in word problems when the situations of the problems are meaningful to them.

Another set of behaviors in regard to arithmetic was stated by Garton in her book entitled, Teaching the Educable Mentally Retarded. This author claims that the following behaviors are characteristic of the educable mentally retarded. These include: low transfer of learning, low abstract thinking ability, poor observation and comprehension of details and situations, slow absorption of facts, little initiative, and lack of ability to concentrate.[1] With these characteristics in mind, these students must be taught in such a way that they can understand, and this understanding can in turn be related to real life experiences.

Other research has indicated that these students perform up to mental age expectancy on computational skills, and functional areas such as time and money.[2] They do poorly on situations that involve concept development and

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[1]Garton, Teaching the Educable Mentally Retarded, p. 220.

[2]Frank Hewett, Education of Exceptional Learners (Boston, Allyn and Bacon, Inc. 1974), p. 367.

reasoning. These students may engage more often in elementary means of obtaining an answer, like counting of fingers, than their normal peers. Unlike their normal peers, these students do only half as much academically as their normal peers. Therefore, it is essential that what they do learn is beneficial to their everyday life.

Arithmetic That Is Needed

Based on the characteristics of the educable mentally retarded student, his arithmetic should be taught in small step sequences which are designed to produce success. This student has to obtain a level of success at his tasks or he becomes easily frustrated. Another type of activity is suggested for this student; this activity is called individualized instruction. He is given instruction based on his particular level of achievement and knowledge. He doesn't have to keep up the pace with his fellow students but he achieves based on his own ability and interest. Other activities include small group involvement. The student should also be able to work with concrete materials. It is suggested by researchers that physical activity be a part of these students' instructional experiences.[1] By physical activities,

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[1]Kenneth Lovell, The Growth of Basic Mathematical and Scientific Concepts in Children (London: University of London Press, 1961), Leo Brueckner, Foster Grossnickle and John Reckzeh, Developing Mathematical Understanding in the Upper Grades, (Philadelphia: J. G. Winston, Co., 1957), Zoltan P. Dienes, Building Up Mathematics, (London: Hutchinson Educational, 1960) cited by Austin Connolly, "Research in Mathematics Education and The Mentally Retarded", Arithmetic Teacher 20, (Oct. 1973) p. 495.

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researchers refer to activities where the students actually move things with their hands such as concrete materials. The use of concrete action-oriented teaching aids by students is supported by Piaget, Bruechner, Grossnickle, Reckzeh, and Dienes.[1] In the arithmetic activities, reading should be kept at a minimum. Realizing the type of student one is working with, the standards of evaluation should be reasonable, based on the capabilities of the student. The diagnostic and evaluative techniques should be used frequently. This should be constantly going on so that progress may be noted and also areas of remediation. The diagnostic and evaluative techniques enable educators to make judgements as to what the student knows, doesn't know, and needs more work on.

Another suggestion is brought about concerning the arithmetic of the students. It has been suggested by Burns that these students be taught arithmetic methodically.[2] They need frequent review and maintenance exercises. The concepts that have to be taught will have to re-introduced almost every year. The review and re-introduction is needed because concepts have to be reinforced through these methods or they have a tendency to be forgotten. The levels of

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[1]Ibid., p. 495

[2]Burns, "Arithmetic Fundamentals for the Educable Mentally Retarded," p. 57.

instruction should, as much as possible, refer to specific experiences and situations to which the student can relate, and therefore get a better understanding of the concepts being taught.

There are three major jobs for the teacher of the educable mentally retarded when it comes to arithmetic. The teacher has to decide, "what is the most important to teach these students, understand the weaknesses of these students in the area of arithmetic, and decide upon methods and materials for instruction."[1] Burns, in his article feels that, based on research, it would appear that the major emphases to be placed in arithmetic are the following:

> "a strong arithmetic readiness program, use of concrete materials and manipulative aids, a variety of activities and procedures for each skill, use of grouping and individualized instruction; following an orderly system and sequence; use of considerable oral and incidental arithmetic; meaning and understanding on numbers as contrasted with mere mechanical manipulation of numbers; use of computational skills in meaningful, life-like, socialized situations; and considerable recurrence, distributive rather than concentrated.[2]

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[1]Ibid., p. 59.

[2]Ibid., p. 59.

Johnson and Mykelbust seem to view the disorders in arithmetic that the educable mentally retarded student may have as stemming from two basic problems. These problems are those in other language areas and disturbances in quantitative thinking.[1] If a student is having problems in the language area such as auditory receptive language, he may have problems in arithmetic. This may be the result of him not being able to profit from the teacher's verbal presentation of the principles. In this area, the student may not be able to understand word problem contexts which are utilized in the spoken instruction that is given by the teacher. If the student has difficulty in reading, he may have some trouble interpreting word problems. He may have difficulty in writing down his answers correctly due to poor visual motor integration.

The other aspect that was mentioned before was disturbances in quantitative thinking. Disturbances in this area may result in the student having problems in comprehending certain mathematical principles. In order for the student to acquire the skill in understanding and using quantitative relations, instruction must begin at the basic, non-verbal

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[1]D. L. Johnson and H. Myklebust, Learning Disabilities: Educational Priciples and Practices, (New York: Grune and Stratton, 1967) cited by Frank M. Hewett, Education of Exceptional Learners, (Boston: Allyn and Bacon, Inc., 1974) p. 367.

level. The principles of quantity, order, size, space, and distance must be taught. Connolly found that the developmental sequence proposed by Piaget, that was presented earlier in this study, is relevant to mentally retarded individuals also. The Piaget-type tasks, such as the conservation of quantity, are a function of the mental age of these children. According to Piaget, the notion of numbers, along with other mathematical concepts, are not learned just from teaching.[1] He feels that to a large degree, these ideas are developed by the child himself. The true understanding of the concepts involved comes about with the mental growth of the child. Piaget also feels that, if a concept is introduced prematurely, the learning that takes place is only verbal; true understanding occurs as the child grows mentally and is at the mental age to understand the concept. A child needs to understand the concept. A child needs to understand the principle of conservation of quantity before being able to develop the concept of numbers.

The work of Piaget has certain implications for the teacher engaged in the task of teaching elementary school mathematics. Some implications are as follows:

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[1]Jean Piaget, "How Children Form Mathematical Concepts". Scientific American 189 (Nov. 1953) p. 74.

1. The child's mental growth advances through qualitatively distinct stages. These stages should be looked upon when planning the curriculum.

2. Test the student to be sure he has mastered the prerequisite for that concept before introducing a new concept.

3. The pre-adolescent child makes typical errors of thinking based on his stage of mental growth. The teacher should try to understand these errors.

4. The teacher can help the child to overcome errors in his thinking by providing experiences to show the errors and ways to correct the errors.

5. The student in the pre-operational stage has a tendency to fix his attention on one variable and neglect the others. The teacher should help him overcome this by providing many situations so he may explore the influence of two or more variables.

6. Teachers should teach pairs of inverse operations in arithmetic together because a child's thinking is more flexible when it is based on the reversible operations.

7. Mental growth is encourged by experience of seeing things from many different points of view.

8. Physical action is one of the bases of learning. To learn effectively, a child must be a participant in the events.

9. There is a lag between perception and formation of a mental image. We can reinforce the developing mental image with frequent use of perceptual data.[1]

Cruickshank, in his research on mentally retarded youngsters, found results that were similar to those that were mentioned before. He found that the mentally retarded youngster was inferior to mental age normal peers in:

1. Ability to solve abstract and verbal problems.
2. Ability to solve concrete problems.
3. Their understanding of the operations to solve a problem.
4. Their ability to isolate pertinent information from a body of given data.
5. Their work habits which are characterized by carelessness and immaturity.[2]

Bower did a study on mentally retarded and normal children in which a comparison of their arithmetic competencies was made. A field test version of Key Math was used. He

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[1]Vincent Glennon and Leroy Callahan, Elementary School Mathematics: A Guide to Current Research, 3d ed. (Wash. D.C.: Association for Supervision and Curriculum Development, NEA, 1968) p. 16.

[2]W. M. Cruickshank. "A Comparative Study of Psychological Factors Involved in the Response of Mentally Retarded Children Ages Thirteen through Sixteen (Ph.D. dissertation, University of Michigan, 1946) cited by Austin Connolly, "Research in Mathematics Education and Mentally Retarded" Arithmetic Teacher 20 (Oct. 1973) p. 492.

found that the performance of the mentally retarded was inferior to that of the normal students with the same chronological age. However, when the mentally retarded were compared to normal students of similar mental age they were superior in certain areas. These areas were multiplication, division, money, time, and calendar. The normal youngsters were superior to the mentally retarded in addition, subtraction, numerical reasoning, and measurement. The results of this particular study implied that mentally retarded students perform best on computational and functional areas of arithmetic, weaknesses in areas of arithmetic requiring verbal mediation and weaknesses in work habits typified by careless computational errors, following directions, and organizing their work.[1]

There were three instructional practices or approaches suggested to be used in the arithmetic instruction for the mentally retarded.[2] According to Connolly, one approach stresses language and verbal information processing. Another is known as the manipulative and discovery approach. The third approach is the individualized

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[1]N. Bower. "A Comparison of Arithmetic Competencies by Mentally Retarded and Normal Children" (Master's Thesis, University of Missouri, 1970) cited by Austin Connolly, "Research in Mathematics Education and Mentally Retarded", p. 493.

[2]Austin Connolly. "Research in Mathematics Education and Mentally Retarded," Arithmetic Teacher 20 (Oct. 1973) p. 495.

instruction approach. Bereiter and Engelmann place emphasis on the approach that stresses language.[1] They feel that language is the cornerstone of math instruction. The idea that language is central to academic learning is the premise for this approach. With this approach, emphasis is placed on learning to manipulate and interpret arithmetic statements based on consistent rules. The language of arithmetic is looked at.

The second approach, manipulative and discovery approach, looks at learning through concrete action-centered learning materials. These materials provide the students with experiences so that abstractions and concepts can be understood. The third approach, individualized instructions, may incorporate some of the elements of the other two. This approach is based on frequent diagnostic assessments. The student works on a series of tasks based on his own rate. His performance on the previous task dictates what the next set of tasks will be. Based on the three approaches presented, Connolly suggests a teaching arrangement where the approaches are combined. This arrangement is as follows:

> Step 1. The concepts are introduced. Explanation of the subject matter has to be in general terms. The activity is related to the child's past experience

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[1]C. Bereiter and S. Engelmann. Teaching Disadvantaged Children in the Preschool (Englewood Cliffs, N.J.: Prentice-Hall, 1966) cited by Austin Connolly, "Research in Mathematics Education and Mentally Retarded:, p. 495.

and skills.

Step 2. Manipulative materials are provided to be used in groups and by individuals.

Step 3. The child has to orally summarize the concept he has learned.

Step 4. Individual materials are provided for the child. These materials help to verify that the child has understood the concept and can apply it.[1]

Manipulative Devices

Under the umbrella of manipulative devices, we find the Cuisenaire rods. Manipulative devices are those concrete materials, which when handled by the student, enables him to attain the objective or objectives that have been identified. Since Cuisenaire rods are manipulative devices, it is appropriate that there be a focus on manipulative devices presented here. It has been noted that manipulative devices are essential to the instruction program for arithmetic. According to Van Engen, "reactions to the world of concrete objects are the foundations from which the structure of abstract ideas arises."[2]

According to certain researchers, manipulative devices seem to be good devices to use in the learning of

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[1]Connolly, "Research in Mathematics Education", p. 496.

[2]Henry Van Engen. "The Formation of Concepts", In The Learning of Mathematics: Its Theory and Practice (Washington D.C.: The National Council of Teachers of Mathematics, 1953) p. 69-98, cited by Emil Berger ed. Instructional Aids in Mathematics (Wash., D.C. Nat'l Council of Teachers of Mathematics, Inc., 1973) p. 302.

arithmetic.[1] In research conducted by Dienes, it was understood that the organism seems to wish to explore and manipulate the environment. By doing this, it is able to predict how the environment is going to respond. Adler, a psychologist, feels that physical action is one of the basics of learning. Here again, we see the need for the child to actively participate in the learning activity in order for the learning to occur. Gagné feels that instruction needs to be fundamentally based on the stimulation that is provided by objects and events. He also feels that a child can learn better, especially arithmetic, when he has an abundance of opportunities to manipulate physical objects. These objects are the stimuli, through which concepts are formed. Such learning theories as Piaget's, Bruner, and Flavell's agree that the performance of internal operation accurately is increased by the experience one gets with concrete materials.

Much of the research concerned with manipulative material is inconclusive. However, there are those who show positive results toward the use of manipulative devices. Harvin, for example, used questionnaires in his study and found that the frequency of the use of such materials appears to be a contributing factor in achievement in mathematics. Adkins and Suddeth found in their study that

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[1]Berger, Instructional Aids in Mathematics, p. 302.

their is a tendency to use more instructional materials in primary grades for motivation, influence of attitude, and the purpose of discovering relationships. Sole, on the other hand, found that the use of a variety of materials does not produce better results than a single device if both of these methods were used for the same amount of time. He feels that regardless of what materials and how many materials are used it is the amount of time spent that is important. If more time is spent, achievement is improved.

According to Bernstein, there are certain principles on the selection and use of manipulative materials. They are:

1. There should be correlation between the operations carried out with the device and those carried on in doing the mathematics with pencil and paper.

2. The aid should involve some moving parts so that it illustrates the mathematics principles that are involved.

3. The device should exploit as many senses as possible.

4. The student should have his own device along with ample time to use it.

5. Learning may proceed from using physical models to using pictures to using only symbols.

6. The use of manipulative devices should be permissive rather than mandatory to the child.

7. The device should be flexible and have many uses.[1]

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[1]Allen Bernstein, "Use of Manipulative Devices in Teaching Mathematics," The Arithmetic Teacher, 10 (May 1966) p. 280.

The purpose of manipulative devices is to convey an idea. A student is aided in understanding a concept by using a manipulative device. With this purpose in mind, one has to be careful in choosing the manipulative device. Hamilton suggests some characteristics that may help in the decision. The Characteristics are:

1. The outcomes and organizations of the device must not be obscure.

2. Variety is provided.

3. The device is simple to operate.

4. The device should be easy to assemble and store.

5. The parts should not be easily lost.

6. The device should encourage communication of some sort.

7. The device should not be an end in itself.[1]

Other characteristics that should also be included are durability, attractiveness, simplicity, size, and cost of the device. The device should be able to withstand regular use by children. It should appeal to children and be designed so that it is easily manipulated by them. The cost should not provide a true embodiment of the mathematical concept to be explored along with a basis for abstraction.

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[1] E. W. Hamilton, "Manipulative Devices", The Arithmetic Teacher, 13 (October 1966): 462.

There are certain uses of manipulative devices as proposed by Reys. Manipulative devices are used to vary instructional activities, provide experiences in actual problem-solving situations, provide concrete representations of abstract ideas, provide an opportunity for students to discover relationships and formulate generalizations, provide active participation by the pupils, provide for individual differences found in pupils, and increase motivation.[1] The manipulative device, with all its uses, should be selected wisely. It should not be a substitute for teaching, but a convenient aid in the process of learning. The device should be introduced to the student in such a way that he feels comfortable using it and be able to ask questions about it, along with making errors and even correcting these errors for himself.

Many teachers have accepted the use of manipulative devices and visual aids in teaching of arithmetic. There was a study conducted which looked at the effectiveness of selected materials for teaching arithmetic. This study used three groups of first graders. It took place in Oak Park, a Michigan school, during the full school year of 1960-61. They used three sets of manipulative materials One group used a commercial set of devices called <u>Numberaid</u>.

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[1]R. E. Reys, "Consideration for Teachers Using Manipulative Materials," <u>The Arithmetic Teacher</u>, 18 (Dec. 1971): 555.

The second group used a set that was selected by the teacher, and the third group used an inexpensive set of materials. The children in the study were given achievement tests and attitude surveys. Group one's materials included a Numberaid abacus for each pupil, a demonstration model, workbooks, and guidebooks for parents. The cost per pupil was five dollars. Group two's materials consisted of ten bead factfinders, fifty plastic sticks, ten round cardboard discs, and hand-operated adding machines. The materials were used to supplement the text book. The cost per pupil was one dollar. Group three's materials consisted of text books and materials found in the first grade. These were supplemented by homemade materials of the teachers. The three groups engaged in certain aspects of mathematics such as counting, addition of whole numbers, subtraction, along with some multiplication and division. The study, which involved 654 students, found that there were no significant differences in the groups, based on arithmetic computation, reasoning, and total achievement when the mean score for I. Q. subgroups 125 and above and 99 and below were used. There were no significant differences in attitude. Based on these data, the researchers felt that expenditures for manipulative devices don't seem justified.[1]

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[1]Hardwick W. Harshman, David W. Wells, and J. Payne, "Manipulative Materials and Arithmetic Achievement in Grade I. The Arithmetic Teacher, 9 (April 1962): 191.

Another study that involved manipulative devices was conducted in the Santa Clara County. Three elementary schools were chosen, one second grade class from each school. A series of twenty lessons for thirty-five minute periods were conducted in the three modes. The modes were manipulative (M), pictorial ()), and abstract (A). The instructional period began on April 29, 1972 and ended on April 31, 1972. Their knowledge of the concept was measured by an investigator-prepared instrument. The test measured the students' ability to use multiplication concepts abstractly, the use and interpretation of numerals, operations and relation of symbols, and facility with mathematical sentences. There were no significant differences between the manipulative and pictorial groups in their ability to affect the children's concept formation in beginning multiplication.[1] This study gives no evidence to prove that concrete materials contribute more to concept formation than pictorial materials.

Finely conducted an experiment with fifty-four educable mentally retarded students. The students were presented with twenty problems in the concrete, symbolic, and pictorial modes. The concrete mode used money and

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[1]Lloyd Scott and Herman Neufeld, "Concrete Instruction in the Elementary School Mathematics: Pictorial vs Manipulative," School Science and Mathematics 76 (January 1976): 70.

actual objects where the students were tested individually. Problems involving pictures were administered as a group. the symbolic aspect was administered to the group. It was found that the educable mentally retarded performed best on the symbolic test and worst on the concrete test.[1] The experimenter wondered that the concrete test was given individually there may be a possibility that the student felt more at ease in the group than when he was singled out. Along with this, there was a lack of control of important variables such as the teacher variable, and the effects of practice.

In light of what has been previously mentioned, Smith feels that practical application should be stressed in teaching mathematics to the mentally retarded. He also feels that such instruction as money value should start with the manipulation of real things.[2] Here the need for manipulation of the concrete is emphasized.

Weber studied the effect of reinforcement of mathematics concepts. This was done with first graders. She used paper and pencil follow-up activities or manipulative materials for follow-up activities. A standardized

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[1]Carmen Finely. "Arithmetic Achievement in Menally Retarded Children: The Effects of Presenting the Problem in Different Contexts," American Journal of Mental Deficiency, 67 (Sept. 1962), p. 281-86 cited by Glennon, Elementary School Mathematics, p. 46.

[2]Frank Hewett, Education of Exceptional Learners (Boston: Allyn and Bacon, Inc., 1974) p. 369.

test was given which showed that there were no significant differences in the activities providing reinforcement. However, there is a trend that favored the groups using the manipulative materials. These students scored significantly higher on the oral test of understanding.[1]

Lucas conducted a study with first graders. His study involved attribute blocks. The purpose of the study was to look at the effects of attribute blocks on the first graders. His results were quite positive. They showed that the children illustrated a greater ability to conserve number and conceptualize addition-subtraction relations.[2] Based on what has been mentioned, there seems to be a positive feeling toward manipulative devices and their place in mathematics instruction.

Studies Involving The Cuisenaire Rods

There have been some studies done with the Cuisenaire rods. Some of the studies involved normal children, while others involved the educable mentally retarded

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[1]Audra W. Weber, "Introducing Mathematics to First Grade Children: Manipulative vs Paper and Pencil," Dissertation Abstract International, 30A (Feb. 1970): 3372-73 cited by Joseph Payne, ed., Mathematics Learning in Early Childhood (Virginia: Nat'l Council of Teachers of Mathematics, Inc., 1975), p. 53.

[2]Joseph Payne, ed. Mathematics Learning in Early Childhood, (Virginia: Nat'l Council of Teachers of Mathematics, Inc., 1975) p. 53.

children. Those studies which involved the normal children will be reviewed.

Haynes conducted a study where the effectiveness of the Cuisenaire method was compared with a conventional method of teaching multiplication. Five third grade classes were in the study. The conventional method was that method set forth in the book, The New Discovering Numbers. The Cuisenaire method was based on the contents found in Mathematics With Numbers in Color: Book A and A Teacher's Introduction to the Cuisenaire-Gattegno Method of Teaching Arithmetic. A total of 106 students were in the study. One teacher in each of the two schools taught a control and a Cuisenaire group for six weeks. A third teacher taught using only the conventional method. The students were given several tests, including the Metropolitan Achievement Tests and Primary Mental Abilities Tests. This study showed that the Cuisenaire method was no more effective in teaching multiplication to third graders than the conventional method.

Another study was conducted by Crowder. His study looked at the effectiveness of the Cuisenaire method as compared to the conventional method in teaching arithmetic to first graders. An arithmetic achievement test and an inventory were used to measure the outcome of the study. Three hypotheses were tested: 1) The experimental group's arithmetic achievement was significantly greater than the

control group's achievement, 2) Sex seems immaterial to the ability to learn arithmetic in grades one and three. 3) There would be very little difference in achievement between upper and middle groups classified by socioeconomic status. The results of the study were:

> The pupils using the Cuisenaire method learned more conventional subject matter, more mathematical concepts and skills than those that were taught using the conventional program. The average and above average students profitted most from the Cuisenaire method. Sex did not matter in terms of achievement. Socioeconomic status is an important factor in ability to learn arithmetic in first grade.[1]

It seems as though the use of objective materials helps learning of arithmetic in first grade.

Lucow conducted a study in which he used third graders. There was a Cuisenaire group and a control group in each of the eight schools. A pretest and a posttest were administered. The experimental period lasted six weeks, starting January, 1962. It was noted that the pretest revealed that the Cuisenaire group was ahead of the control group. At the end of the experiment the following conclusions were made:

> Cuisenaire method proved to be effective for teaching third grade mathematics and other methods of instruction are also effective.

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[1]Alex Blecher Crowder, Jr., "A Comparative Study of Two Methods of Teaching Arithmetic in the First Grade," Dissertation Abstracts, (Ph.D. dissertation, No. Texas Univ., 1965)

> There was evidence that the Cuisenaire method operates better in a rural setting than an urban setting.
>
> The Cuisenaire method operates better on high I. Q. and middle I. Q. in a rural setting, but not much better with low I. Q.
>
> There is a slight indication that girls take to the Cuisenaire method better than boys.[1]

The experimenter felt that the results that favored the Cuisenaire group could be based on the presence of over-aged pupils, repeaters, and deviants in the control group and the differences in mental set toward multiplication and division (The process new to control group but familiar to the Cuisenaire group). In his summary of the study, Lucow feels that the Cuisenaire method is a good one and it should be added to the third grade teacher's repetoire. He also feels that the children should be taught by whatever method they respond to. With the fact that children have individual differences, it is felt that a teacher should not limit himself to one method of instruction.

Another investigator, Passy, wanted to find out the effects of the Cuisenaire method on reasoning and computation. The program was limited to the first three grades and Kindergarten. 1,200 children at each grade level participated. They were tested in May, 1962 with the Stanford Achievement

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[1]William Lucow, "An Experiment with the Cuisenaire Method in Grade Three," American Educational Research Journal, 1 (May 1964): 166.

Test, Elementary Battery and grade scores on arithmetic reasoning and computation. There were three groups. One used the Cuisenaire method in modified elementary curriculum. The second group used a meaningful arithmetic program (not Cuisenaire). Group three was drawn from the pre-Cuisenaire third grade, in the first group. The first group were third graders. This study revealed that the third graders who used the Cuisenaire method achieved significantly less at the .05 level on the arithmetic subtest of the Stanford and Elementary Battery than the other two groups.[1]

A study was conducted to look at the effects of the Cuisenaire method on the teaching of first grade mathematics. The study began in October 1961 where nine classes of first graders in three schools were selected. Four classes in one school used the Cuisenaire method. Five classes, two in one school and three in another, used the traditional method. Both groups were given a series of three tests at the end of the school year. At the beginning of the 1962-63 school year, the students in the experimental group were assigned to three second grade classes and the controls were assigned to five second grade classes. The instruction lasted twenty-five minutes per day. At the end of the

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[1]Robert Passy, "The Effects of Cuisenaire Materials on Reasoning and Computation," The Arithmetic Teacher 10 (Nov. 1963): 440.

1962-63 school year similar tests were given. Hollis, the researcher, made these conclusions: at the end of the first year and second year, the Cuisenaire method taught traditional subject matter as well as the traditional method and the students in the Cuisenaire group acquired additional concepts and skills to those taught in the traditional programs.[1]

There was a study made in Scotland and England by Brownell. The study was looking at the effectiveness of three programs. These programs were the conventional method (experiences in grouping and use of discrete objects), the Cuisenaire method, and the Dienes method (use of multi-based blocks). The study was concerned with children who had three years of schooling. Schools were selected in Scotland and England (forty-five schools with 1,430 children). There were comparisons made between the Conventional and Cuisenaire groups in the Scottish school. The three programs were looked at in the English schools. The study revealed that the Cuisenaire method was more effective than the Conventional method in developing meaningful mathematical

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[1]Loye Hollis, "Cuisenaire-Gattegno Method with a Traditional Method, A Study to Compare the Effects of Teaching First and Second Grade Math," School Science and Mathematics 65 (Nov. 1965): 685.

abstractions. However, in the English study, the Conventional method ranked the highest in overall ranking for effectiveness in promoting conceptual maturity. The Dienes and Cuisenaire methods ranked equal in this aspect. In this study, there were uncontrolled variables. These were the differences in quality and amount of instruction, the pace of the instruction, and the objectives involved. Although the results seem to point to the Cuisenaire as being more effective, Brownell raised the question that the differences in the results may be due to the skill and enthusiasm portrayed by the teachers and not in the material itself.[1] It may be that the teacher was enthusiastic about teaching something new and this raises the problem of quality of teaching. This variable is very difficult to account for. It seems as though in the English schools, where the novelty of the Cuisenaire method no longer intact, the Conventional method seems to have been better.

An experiment was conducted in Vancouver in the Fall

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[1]Callahan, Elementary Mathematics, 4th ed., p. 15.

of 1957. It consisted of one experimental and one control group in five schools in Vancouver. Four sets of Cuisenaire rods were supplied to the experimental groups. Instruction began in October. The time period of the instruction was twenty minutes a day (ten minutes for teaching and ten for seatwork). They were given four types of tests: A Detroit beginning first grade intelligence test (September, 1957), an initial survey test in number work (January, 1958), a terminal test based on the course of the numberwork for first grade (June, 1958), and survey test for content taught with Cuisenaire materials (June, 1958). The study was conducted during the 1957-58 school year with first graders. The mean score of the Detroit test was higher for the experimental group, but it was not statistically significant. There were significant difference noted in the mean test scores on the survey test of the content taught with the Cuisenaire materials. The performance of the experimental group (Cuisenaire) was superior. This shows that the Cuisenaire method may be more effective than traditional ones with bright and slow children. The conclusions of the study are as follows:

1. There was no significant difference between the experimental and control groups in rate of learning in Grade 1 numberwork.

2. The experimental group surpassed the control group

in terms of the facility with more complex combinations of whole numbers and common fractions.

3. There were no significant differences between the groups on their performance on two problem items that required reading and reasoning.

4. In terms of gains in scores on tests of basic Grade 1 numberwork, the effectiveness of the method of instruction is independent of whether the group is bright or slow.[1]

The following year a second experiment was conducted in Vancouver. The results of this experiment were similar to the one conducted before. An analysis of variance revealed that there was a highly significant relationship between ability and achievement of the groups. There was also a difference in achievement of the groups but it was not significant. Again, the effectiveness of the method was shown to be independent of whether the group is slow or bright.

An experiment was conducted in Saskatchewan during the 1958-59 school year. It was conducted for ten months with 461 in the experimental group and 263 in the control group. A series of tests were administered to the students

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[1]Ivy Hinchliffe. "A Report on an Experiment to Evaluate the Effectiveness of Two Different Methods of Teaching Arithmetic at the Grade One Level," (Thesis, University of Manitoba, 1961) p. 46.

involved in the study. The tests were: power tests administered three times, a special test which had more problems with fractions, and the Pintner Ability Test. After the first year, it was continued for the 1959-60 school year. The experiment involved first, second, and third graders. Based on the results which showed that the Cuisenaire group got higher scores on the tests, it seems as though the method is an effective one. The study was conducted by the Saskatchewan Teachers Federation. A questionnaire was administered to the teachers involved in the study. This subjective data leaned positively toward the Cuisenaire rods. It seems as though the teachers and the students enjoyed working with the rods. After this experiment, another experiment was suggested to be conducted the following year. In 1962 the Saskatchewan Teachers Federation put together a report on the Cuisenaire method. The report consists of principles and procedures of the method. Suggested activities to be used in grades one, two, and three are also in the report. It seems as though the Saskatchewan Federation of Teachers felt that the Cuisenaire method is a good one for teaching mathematics.[1]

Karatzina and Reinshaw conducted a study in Edinburgh during the 1957-58 school year. The experimental

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[1]Saskatchewan Teachers Federation. "Cuisenaire: A Sound Approach to Teaching Mathematics," (Saskatchewan Teachers Federation, 1962) p. 3.

period was eighteen months in which there were forty boys in the group, and fourteen girls and twenty-four boys in the control group. There were three tests administered. They were the Thurstone Primary Mental Abilities Test, Moray House Picture Test, and the Schonell Diagnostic Arithmetic Test. The mean scores of the tests were close, however, there were no statistically significant differences found at the .05 level.

Hinchliffe conducted a study which involved the Cuisenaire method. The study was conducted during the 1959-60 school year in Manitoba. It lasted for ten months. There were 230 first graders in the experimental group and 189 in the control group. This study, like the others, was based on the results of tests that were administered. The experimental group was taught using the Cuisenaire method and the control group was taught using the Living Arithmetic Series. Along with the objective data, subjective data were collected through questionnaires about the advantages and disadvantages of the Cuisenaire method. The questionnaires were given to the teachers and administrators of the classes that were involved in the study. The results of the study are as follows:

1. There were greater gains in achievement of the experimental group.

2. The experimental group exhibited greater skills in

computation.

3. The measurement of ability was in favor of the experimental group.

4. The questionnaires revealed that there was a positive attitude towards the rods.

5. The questionnaires showed that the skills were developed more quickly and easily.

6. The teachers and students enjoyed using the Cuisenaire method.

7. The Cuisenaire method proved to be a challenge for the brighter students.

8. The teachers suggested using the method more.[1]

Another study was conducted using the Cuisenaire method. This study, unlike the others previously mentioned, involved preschoolers. To be exact, the study used five students, each with an average of three years. The rods were used for three months in an attempt to teach them mathematical concepts through free play. There were four girls and one boy, all of average intelligence. The method used by the researcher, Karen Beard, was one in which the children played with the rods only while they were interested in the rods. The knowledge came about through

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[1]Hinchliffe, "Report on an Experiment to Evaluate the Effectiveness of Two Different Methods of Teaching Arithmetic at the Grade One Level," p. 138.

dialogue concerning the rods. These materials hastened the conceptual communication between the children themselves and between the teacher and the children. The researcher made suggestions and invented games in which the students engaged in. They were tested before and after the treatment period. The tests that were used included an awareness test (made up by the researcher), and the Incomplete Man Intelligence Test. The awareness test contained questions on geometry, naming of colors, counting, reading numbers, and relationships among colored lengths of paper.

The students went through four stages while working with the Cuisenaire rods. These stages were: free play (manipulating the rods any way the students wants to explore the rods), free play with directed activities in which the relationships were observed, free play with mathematical notation, and free play with number introduction and written work.[1] Free play is needed so that an atmosphere of creativity and curiosity is established. The purpose of free play are:

1. To present a situation from which a student can unconsciously learn math.

2. To discover what can be done with the rods and see

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[1]Dagny Karen Beard, "An Intensive Study of the Development of Mathematical Concepts Through the Cuisenaire Method in Three Year Olds," (Thesis, Southern Connecticut State College, 1964) p. 31.

what pattern can be made.

3. To provide the teacher with an opportunity to see during the play, what concepts may be forming in the student's mind based on what he does with the rods.

4. For aesthetic pleasure and enjoyment.[1]

In the three months where sessions were conducted three times a week for a period of forty-five minutes to an hour there were some positive conclusions made. Beard feels that the rods were constructive in helping the children to visualize the concept of number, and such operations as addition and subtraction. It was observed that when the student became familiar with the processes involved, he no longer needed the rods and dropped them voluntarily. The students learned by themselves through the dialogue with the rods, observations of others, the involvement in games and discussion about the rods. The results of the study proved the contention of Cuisenaire developers that algebra precedes arithmetic.[2] In working with the rods, the students look at relationships of the rods (for example, one red rod is twice as long as the white rod). This can be written as W+W=R. Algebra is illustrated here by the use of letters. Later, arithmetic comes into play with the

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[1]Ibid., p. 31.

[2]Ibid., p. 55.

use of letters. Later, arithmetic comes into play with the use of numbers.

The studies concerning the Cuisenaire rods that were mentioned before involved the normal children found in the regular classroom. Callahan also conducted a study. He used retarded children in his study. The study started in February, 1966 and lasted for nine weeks. It involved a class of mentally retarded students (nine) whose age range was between seven and ten, and whose I.Q. score range was less than eighty and as low as fifty-seven. The first few classes were devoted to free play of the rods and getting acquainted with the physical properties of the rods. Then they engaged in identifying colors and assigning letter names to the colors. They were taught to build staircases with the rods and to make equations. This introductory period lasted three weeks. The fourth week was spent on developing the basic understanding of addition, subtraction and multiplication facts from one to five. At the end of the fourth week, a test was adminstered. The next five weeks were spent on establishing math facts of numbers six to ten. At the end of the experimental period, the rods were evaluated based on the structure of the child's ability. There were nine important points that came about by this study. These are:

1. The rods satisfied all areas of the Boston School Document of the course of study for the special classes at

the elementary level. These areas are development of an understanding of number concepts, teach the fundamental processes, application of the four processes to a problem, development of an understanding of measurement, and good work habits.

2. The concepts appear to remain with the student after the rods are removed.

3. The number facts were learned from concrete situations.

4. The rods encouraged the ability to recognize reverse operations (subtraction is the reverse of addition).

5. The commutative property is illustrated in a phyical situation by making trains.

6. The rods provide an opportunity for more advanced math than the coventional method.

7. The results achieved by the rods are better than achievement without the rods.

8. The children do not appear frustrated when manipulating the rods.

9. The rods enabled the students to make discoveries and retain information.[1]

The study seems to have results that are positive in terms

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[1]John Callahan and Ruth Jacobson, "An Experiment with Retarded Children and Cuisenaire Rods," The Arithmetic Teacher, 14 (January 1967) p. 13.

of the effectiveness of the materials. It seems as though these materials work pretty well with the educable mentally retarded.

Suggested Mathematics for the Educable Mentally Retarded

The type of math that was suggested for these students was a comprehensive model which stressed verbal information processing. The model would also stress conceptual development and numerical reasoning. This type of model was suggested by Crawley and Vitello.

When teachers were asked what type of mathematics instruction to be used with EMR students, many of them suggested that these students be given more time in working through their arithmetic assignments in the regular math program. Along with this, the notion of more use of concrete materials is generally accepted by teachers of EMR students.

Kirk and Johnson feel that the context to be taught to these students should be chosen on the basis of the following two principles: 1) content must include the knowledge, skills, and concepts that will be of the most value to these students later in life and 2) methods that are used should be determined by the special disabilities or abilities of these children.[1]

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[1]S. Kirk and O. Johnson. Educating the Retarded Child (Boston: Houghton Mifflin Co. 1951) cited by Callahan Elementary School Mathematics, p. 71

Crawley and Goodman stressed the use of manipulative and pictorial devices in working with educable mentally retarded students. These devices seem to help in the learning situation of these students. There was a demonstration program illustrated by Crawley and Goodman. The results showed an improvement in verbal problem-solving and understanding of the principles involved.

Goodstein, Bessant, Thibodeau, Vitello, and Vlahakos conducted a study which looked at the verb al problem-solving ability of the educable mentally retarded student. They found that the use of pictorial aids helped these students obtain a greater degree of performance in that area.

Others suggest the use of programmed instruction. Some of the benefits that were suggested by the use of programmed instruction were a reduction of time required to obtain a skill and a reduction in negativism and hostility toward the method of instruction. However, there is no clear and consistent advantage of this method over others. As a matter of fact, some researchers such as Smith and Quanhebush found that the students seem to need additional reinforcement besides that given by the machines. Most of the students needed approval by the teacher.

It is a common notion among educators that learning should be meaningful. The teacher of an EMR student has to be able to teach what is essential for the students to know and yet teach it in a way that there are developmental and

practice activities that lead to understanding and application. It has been suggested by Callahan that the teacher of these students pay attention to the principles of development that govern the child's mode of thinking, the principles of cognitive adaptation (conservation, equivalence, and flexibility), and the factors that are related to visual perception.[1]

In light of what has been mentioned concerning what type of mathematics to teach EMR students, it seems as though the majority of what is being taught is the same as what is being taught in the regular class. According to Cawley, "There isn't a single comprehensive arithmetic program that has been developed, tested and validated for use throughout school age range for the mentally handicapped children."[2] He also feels that until such a program is established the educator will not know whether the low achievement is the result of poor instruction, the curriculum, or the limitations of the individual. One of the greatest limitations in terms of searching for a good program for the mentally handicapped (another term for EMR) is the notion that these students are concrete learners. This notion has led to a

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[1]Callahan, Elementary School Mathematics, p. 72

[2]John F. Cawley, "Teaching Arithmetic to Mentally Handicapped Children," Strategies for Teaching Exceptional Children, (Denver: Love Publishing Co., 1972) p. 250.

the present generation, along with future generations, may be able to live comfortably in the world.

CHAPTER III

DESIGN

The study looked at the following hypothesis: There will be a significant difference in the mean of the posttest scores of the students using the Cuisenaire rods as compared to the mean of the posttest scores of those students using the traditional method when adjusted for previous mathematics knowledge and intelligence.

SOURCE

The study involved two primary educable mentally retarded classes. One was at the Ulla Muller Elementary School and the other one was at the Peace Corps Elementary School. Eighteen educable mentally retarded students were involved, ten in the experimental group and eight in the control group. The age of the students ranged from nine to eleven years.

PROCEDURE FOR COLLECTING DATA

The study was quasi-experimental in design. Both the experimental group and the control group were tested prior to the treatment period. At the end of the experimental period, a posttest was administered to both groups. The

test that was used was the Stanford Achievement Test Primary Level II. Only the mathematics computation test of the battery was used. The test items were randomly split in half along with the time for administration of the test. One-half of the test served as the pretest and the other half was the posttest. Test time for both tests was fifteen minutes.

Both the control and experimental groups remained in their self-contained classrooms. The researcher taught both the experimental and control groups during the three week period. Throughout the experimental period of three weeks, the experimental group worked with Cuisenaire rods to solve addition and subtraction problems involving whole numbers. The control group used the Mathematics Around Us textbook for grade two, Publisher - Scott, Foresman and Company, along with workbook exercises and drill exercicises placed on the chalkboard. Both groups were taught each day for a period of three weeks for approximately sixty minutes each day. The mathematics problems consisted of one and two digit addition and subtraction problems.

ANALYSIS OF DATA

The data was analyzed by the use of the analysis of covariance. The I. Q. and pretest scores were used as

the covariates. The posttest scores were the dependent variables and the treatment was the independent variable. There was a comparison made of the pretest and posttest scores of both groups to see if any gains in achievement were made. The T Test was used to analyze this.

CHAPTER IV

THE FINDINGS

The research was conducted over a period of three weeks where two groups of educable mentally retarded students were taught the above mentioned concepts. One group, the experimental group, used the Cuisenaire rods while the other group, the control group, used the traditional method. Both groups were taught by the researcher for approximately one hour, five days a week. Each group was given a pretest at the beginning of the three week period, and a posttest at the end of the period. The posttest scores served as the dependent variable, and the treatment received by each group served as the independent variable. The criterion variable is the posttest, and the control variables are the pretest and the I. Q. scores.

Table I presents descriptive data on pre-and post-test and I. Q. scores of the two groups. It was shown by the analysis of covariance that the null hypothesis cannot be rejected at the .05 level of significance (Table II). There were no significant differences in the means on the posttest scores at the .05 level of significance.

The T Test that was used for both groups showed that the difference between the means of the pre- and post-test

TABLE I

MEANS OF PRETEST AND POSTTEST FOR TWO GROUPS OF EMR STUDENTS IN MATHEMATICS

Method	n	Pretest Means	Posttest Means	I.Q. Means
Traditional	8	51.38	74.25	59.75
Cuisenaire	10	52.90	72.30	64.30
Total	18	104.28	146.55	124.05

TABLE II

ANALYSIS OF COVARIANCE FOR ACHIEVEMENT DIFFERENCES BETWEEN TWO EXPERIMENTAL MATHEMATICS GROUPS CONTROLLING FOR PRIOR MATHEMATICS ACHIEVEMENT AND INTELLIGENCE

Source of Variation	Degrees of Freedon	Sum of Squares	Residuals Mean Square
Between	1	56.6	56.6
Within	14	2210.0	157.86
Total	15	2266.6	

$$F = \frac{\text{mean square (between)}}{\text{mean square (within)}} = \frac{56.6}{157.86} = .359$$

CHAPTER V

DISCUSSION AND RECOMMENDATIONS

This study attempted to show the effectiveness of the Cuisenaire Rods with educable mentally retarded students. It was found that there was no significant difference in the two methods employed. There was achievement in both groups but there was no significant difference of one method over the other. Therefore, the method employed did not make much of a difference in the achievement.

Throughout the three week period, there are certain observations made by the researcher that needs to be noted here. In the review of the literature, certain characteristics of the educable mentally retarded were mentioned. Some of these characteristics were observed by the researcher. These characteristics are: 1) The use of fingers in counting, 2) They made a lot of mistakes, 3) Repetition was needed in order for the concept to be maintained, 4) They have a tendency to forget easily and, 5) They were slow in absorbing facts.

Based on the findings of the study, the following recommendations are submitted:

1. A study similar to this one should be conducted for a longer period of time. A suggested period of two

APPENDIX A

SCORES OF TWO GROUPS (CUISENAIRE AND TRADITIONAL) ON A PRETEST (X_1), A POSTTEST (Y), AND I. Q. (X_2)

SUBJECTS	Y	X_1	X_2
TRADITIONAL			
1	59	41	70
2	65	41	64
3	88	53	49
4	76	41	41
5	100	88	71
6	71	71	72
7	76	47	59
8	59	29	46
TOTAL	594	411	478
CUISENAIRE			
9	82	59	68
10	94	53	55
11	71	47	67
12	65	59	55
13	47	41	54
14	76	82	66
15	65	29	74
16	59	59	69
17	76	41	68
18	88	59	67
TOTAL	723	529	643
GRAND TOTAL	1,317	940	1,121

APPENDIX B

PRETEST

Samples

A. 3+1=☐ 3 4 5 2 NH O ● O O O	B. ☐ -2=1 5 4 2 3 NH O O O O O	C. 2 3 +1 3 7 5 4 NH O O O O O
1. 2 +7 8 9 10 7 NH O O O O O	2. 9 +6 17 14 16 13 NH O O O O O	3. 8 +6 14 3 1 2 NH O O O O O
4. 15 -9 6 14 5 7 NH O O O O O	5. 7 8 +8 22 23 24 21 NH O O O O O	6. 13 -6 13 10 7 5 NH O O O O O
7. 109 -43 143 66 103 43 NH O O O O O	8. 11-4=☐ 6 8 7 15 NH O O O O O	9. ☐+3=11 9 14 7 8 NH O O O O O
10. ☐-7=6 7 13 1 11 NH O O O O O	11. ☐ -9=9 17 3 0 18 NH O O O O O	12. 83 +46 139 21 119 129 NH O O O O O
13. 127 -63 64 144 44 180 NH O O O O O	14. 108 -54 104 17 154 54 NH O O O O O	15. 98+9=☐ 105 107 26 917 NH O O O O O

Note: The meaning of the symbols below:
> is greater than
< is less than
= is equal to

Samples A. 5 O 3+1 B. 4 O 2+3 C. 2+4 O 3+3	>O ● >O >O	<O <O ● <O	=O =O =O ●
16. 7+8+6 O 8+5+7	>O	<O	=O
17. (7+6)+8 O 6+(6+8)	>O	<O	=O

APPENDIX C

POST-TEST

Samples A. 3+1=□ 3 4 5 2 NH 0 ● 0 0 0	B. □-2=1 5 4 2 3 NH o o o o o	C. 2 3 +1 3 7 5 4NH o o o o o
1. 4 +5 8 9 7 10 NH o o o o o	2. 8 +5 12 13 14 11 NH o o o o o	3. 7 -5 1 3 2 12 NH o o o o o
4. 9 7 +8 21 23 22 25 NH o o o o o	5. 43 +63 116 96 16 NH 106 o o o o o	6. 79 -52 26 23 25 28 NH o o o o o
7. 3+4=□ 6 8 7 9 NH o o o o o	8. 2+□=8 10 5 6 7 NH o o o o o	9. □+5=12 7 17 8 6 NH o o o o o
10. 10-□=4 5 7 6 4 NH o o o o o	11. 63 +55 19 128 118 108 NH o o o o o	12. 175 +73 248 148 148 247 NH o o o o o
13. 128 -42 160 86 126 26 NH o o o o o	14. 54+7 = □ 59 16 51 61 NH o o o o o	

Note the meaning of the symbols below:

> is greater than
< is less than
= is equal to

	>	<	=
Samples A. 5 o 3+1 B. 4 o 2 3 C. 2+4 o 3+3	●o o o	o o o	o o o
15. 5+4 o 4+5	o	o	o
16. 11-8 o 12-7	o	o	o
17. (5+9) 3 o 5+(4+9)	o	o	o

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COLLEGE OF THE VIRGIN ISLANDS

GRADUATE PROGRAM

May 12, 1981

IT IS HEREBY RECOMMENDED THAT THE THESIS BY Vernice Williams

ENTITLED The Effectiveness of Cuisenaire Rods on the Instruction of Educable Mentally Retarded Students

BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

MASTER OF ARTS IN EDUCATION.

COMMITTEE

THESIS ADVISOR

FOR THE

FINAL EXAMINATION

CHAIRMAN, GRADUATE STUDIES

COLLEGE OF THE VIRGIN ISLANDS

Graduate Program

Thesis Oral Examination Report

The consensus of the oral examination committee is that:

Student Vernice Williams

Title The Effectiveness of Cuisenaire Rods on the Instruction of Educable Mentally Retarded Students

Date May 12, 1981

A. Passed-unqualified ______________________

B. Passed-conditionally, based on corrections to be made and resubmitted for approval to the thesis advisor

X

C. Passed-qualified, corrections to be made and submitted to the entire committee

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D. Failed ______________________

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Thesis Advisor

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Second Committee Member

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Graduate Council Examiner

COLLEGE OF THE VIRGIN ISLANDS

THE EFFECTIVENESS OF THE CUISENAIRE RODS ON THE INSTRUCTION OF EDUCABLE MENTALLY RETARDED STUDENTS

A THESIS SUBMITTED TO
THE GRADUATE STUDIES COUNCIL
IN PARITIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE
OF
MASTER OF ARTS IN EDUCATION

BY

VERNICE WILLIAMS

ST. THOMAS, VIRGIN ISLANDS

May, 1981

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CHAPTER I

BACKGROUND

The role of educators has been for some time to teach students, whom they come in contact with, the necessary skills that they will need in order to function within our society. Among the goals of education, one of the most important goals is the transition of knowledge to these youngsters. The knowledge should not be just mere facts and information. Education also has the goal of instilling a desire to learn and to achieve in those students who will be the future leaders in our society. With this in mind, it is hoped that they not only are able to function within our society but to contribute in a positive way to the further development and improvement of the world in which we live. Therefore, the best methods for teaching certain skills so that they may be retained for further use in society are being constantly searched for and analyzed in terms of their effectiveness in learning.

It is a common belief that every child has the right to an education. The educable mentally retarded child is no

exception. He deserves an education also. He was unable to fit into the system of education as we know it. However, due to the passage of Public Law 94-142, every handicapped child has been given the right to an education. The child is no longer forced to change so that he fits into the school system; the school system must change so that it fits into the characteristics of the child. The school system must provide the necessary environment along with the materials to insure that these students develop to their full potential. If the school system does not comply with the law, it can be held liable.

Educable mentally retarded students are considered as part of the group called handicapped or exceptional children. They need to be educated so that they can function within our society and not be considered inferior. One of the most important areas to acquire enough understanding is in the area of mathematics. A knowledge of mathematics is needed in practically every facet of today's society. Mathematics helps us to know the number of days in a month, the amount of any object we may have at a set time, and the amount of money to spend, invest, or save. Not only is the knowledge of mathematics helpful, it has set purposes.

Berger lists the purposes of mathematics as follows:

ulative devices enable the student to actually operate, hold, and really investigate what is placed before him to conceptualize. According to Piaget, learning proceeds through stages.[1] One must start with the concrete level, then to semi-concrete, then to abstract. Here again, we see the need for concrete or manipulative objects for concept learning.

There has not been much research done on the skills of educable mentally retarded students in the area of mathematics. However, the research that has been done shows that certain characteristics are found in the educable mentally retarded students that are typical of that population in terms of mathematics. Glennon summarizes these characteristics as follows:

1. They are retarded in the area of arithmetic vocabulary.
2. They are inferior to normal students in the ability to solve abstract verbal problems.
3. They are better at solving concrete problems than at solving abstract problems.
4. They have less understanding of the processes to be used in a problem situation and are more apt to guess at processes than normal students.
5. They are more careless than normal students at their work, use more immature processes, and make more technical errors.

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[1]Jean Piaget, "How Children Form Mathematical Concepts", Scientific American 189 (November 1953), p. 74.

6. They are less successful in differentiating extraneous material from needed arithmetical facts than normal children.

7. They do equally well with word problems and mechanical operations if the instruction is meaningful to them.

8. They have little concept of time and sequence.

9. They do better with addition and subtraction and need more emphasis on multiplication and division.

10. Arithmetic readiness is even more important to the educable mentally retarded students than to normal students.[1]

With these characteristics, it has been found that this group of students seem to work up to mental age expectancy in mathematical fundamentals. However, this does not occur in arithmetic reasoning in which there is reading and problem-solving. The educable mentally retarded student does develop quantitative concepts in the same order and stages as normal children do. Although they develop these concepts, they are developed later. The concepts are acquired through teaching and maturation. When students are drilled to perform advanced Piagetian-type quantitative tasks, they may appear to have the skills, but they do not really understand the concepts involved. Understanding of the concepts comes

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[1]Vincent Glennon and Leroy Callahan, Elementary School Mathematics: A Guide to Current Research, 3d ed. (Washington, D.C.: Association for Supervision and Curriculum Development, NEA, 1968), p.45.

at a later time after adequate intellectual growth has taken place.[1] According to a number of researchers, concrete materials for teaching mathematics seems to work well with these students. In a similar fashion, it seems as though they learn a particular concept much better when it is presented in the concrete mode. An example of a concept would be addition. In the concrete mode, the child is given two blocks and then two more blocks and asked how many there are all together. The concrete mode is represented through the blocks. The learning style of the educable mentally retarded student is one that encourages him to actually see, hold, and manipulate the material at hand so that it will help him understand the concept. They learn through concrete representation of the subject matter.

Cuisenaire rods emphasize this learning style. They are colored rods that range from one centimeter to ten centimeters long. The student is able to visualize and manipulate the rods. He is able to internalize the concept and gain a better understanding of mathematics by using the rods. These rods were named after Georges Cuisenaire, who invented them. He was a schoolmaster in Belgium. It was because one of his students had problems in understanding mathematics that he started experimenting with pieces of wood. The pieces

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[1]Lloyd Dunn, ed., Exceptional Children in the Schools, (New York: Holt, Rinehart, and Winston, Inc. 1973), p. 148.

were painted various shades of the primary colors, one in black, and the smallest size was not painted. The students in his classroom experimented with them. They soon gained confidence and did mathematical operations with them. Caleb Gattegno was the person responsible for making the rods popular by promoting them abroad. According to Caleb Gattegno, "Color is a factor that is accessible to the minds of almost all humans. Its shades and contrasts can act as a sign to substitute for the abstract notion that it is proposed to attain."[1] It seems as though color would catch anyone's eyes. After this attention is obtained, maybe some learning can be nurtured. The colors for the Cuisenaire rods are based on the following system:

1. All the multiples of two contain the color red, (2cm- red, 4cm- purple, 8cm- brown)

2. All the multiples of three contain the color blue, (3cm- light blue, 6cm- dark green, 9cm- blue)

3. All the multiples of five contain the color yellow, (5cm- yellow, 10cm- orange)

4. Number one was uncolored; number seven was considered the outcast and was colored black.

The Cuisenaire rods focus on the belief that the student sees relationships with the rods in terms of mathematics. These relationships are formulated by the students

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[1] William Ewbank, "The Use of Color for Teaching Mathematics," Arithmetic Teacher, 26 (September 1978): 53.

based on the activities engaged in with the use of the rods. According to the producers of the Cuisenaire rods, the rods capture the attention of students, and this enables students to focus on the task at hand. The philosophy of Cuisenaire rods is one of active teaching. It focuses on seeing, doing, reckoning, understanding, and verification. Based on this philosophy, the student is engaged in manipulation of a concrete set of object, namely the Cuisenaire rods, to work out mathematical operations.

PROBLEM STATEMENT

The question of how to teach the educable mentally retarded has been discussed often. Most researchers feel that they learn best when the subject matter is presented in a concrete manner. In the area of mathematics, the educable mentally retarded have a difficult time conceptualizing the ideas involved. They learn at a slower pace then other students. The particular type of material to be investigated with the educable mentally retarded is the Cuisenaire rods. This material is concrete in its presentation and is used in the area of mathematics. The question of how to best teach the educable mentally retarded mathematics needs to be addressed. How effective are the Cuisenaire rods in helping educable mentally retarded students grasp the concepts of addition and subtraction of one-and two-digit whole numbers? The study will, therefore, look at the effectiveness of Cuisenaire rods on educable mentally retarded students in the area of addition and subtraction of whole numbers (one- and two-digit numbers).

HYPOTHESIS

There will be a significant difference in the achievement of the educable mentally retarded students using the Cuisenaire rods as compared to those using the traditional method.

DEFINITION OF TERMS

1. Educable Mentally Retarded Students A definition as stated by Van Osdol and Shane in An Introduction to Exceptional Children:

   > Mental retardation refers to significantly subaverage general intellectual functioning existing concurrently with deficits in adaptive behavior and manifested during the developmental period.[1]

2. Concrete Materials Those materials which one is able to see, feel, and manipulate.

3. Cuisenaire Rods Colored rods which range in size from one centimeter to ten centimeters long. They are used to improve computational skills and increase mathematical understandings.

4. Traditional Method The use of specified textbook and workbook in teaching mathematics along with the use of drill exercises on the chalkboard with the use of concrete materials when needed.

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[1]William R. Van Osdol and Don G. Shane, An Introduction to Exceptional Children, (Iowa: Wm. C. Brown Company, 1974), p. 57.

LIMITATIONS

The study will be limited to a group of eighteen primary educable mentally retarded students. It will be conducted at two elementary schools on the island of St. Thomas. There will be ten students in the experimental group and eight students in the control group.

SIGNIFICANCE

This study will look at a particular material, the Cuisenaire rods, in terms of its effectiveness with educable mentally retarded students. It is hoped that the study will be of some use to teachers of educable mentally retarded students in their search for appropriate materials for their students.

CHAPTER II

REVIEW OF LITERATURE

In the field of education there is a constant concern about the best method to use in teaching children a particular concept or idea. It has been proven through past studies that children learn in different ways. Another way of stating this is that there are various learning styles related to children. Some children may learn visually, while others may learn better auditorily. The tactile mode may also be another way children learn. There are also those who use a combination of modalities mentioned in order to understand or comprehend an idea. The visual learners learn by actually seeing the illustration of what they are to learn, whether it is represented through a picture, a diagram, or even a model. The learners that learn auditorily use the sense of hearing to grasp an idea or concept. This may be accomplished through the use of such devices as tape recorders, record players, or even simple conversation. The tactile mode involves the sense of touch. With this mode, the student has to actually experience by touching the actual object being discussed.

In view of the modalities, which are ways in which one can learn, there are certain levels of learning or intellectual development. Various educators, along with psych-

ologists, have formulated various levels of learning that they feel a student goes through in order for the concept to be understood. Bruner, for example, has devised three levels of learning that a student goes through.[1] These levels are the enactive, the iconic, and the symbolic. The enactive level, most often refers to the level of concreteness. This level is concerned with concrete objects. The student is presented with the actual object, in order that he may manipulate and look at it to formulate various ideas about the subject matter. After the inactive level, there is the iconic level. This level refers to the pictorial aspect of representation. At this level, picture of objects, diagrams, or sketches are used in helping students to understand a particular concept. The third level is the symbolic level. This level uses symbolism such as words or numbers to capture an idea. At this level, the student no longer needs the concrete presentation or pictorial presentation of a concept, but can understand by just using the symbols. An example of this level would be as follows: when a child is presented with "2+3" he knows what 2 and 3

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[1]Jerome S. Bruner, Toward a Theory of Instruction (Cambridge: Harvard University Press, Belknap Press, 1963) p. 28 cited by Linda Barron, Mathematics Experiences for the Early Childhood Years, (Columbus: Charles E. Merrill Publishing Co., 1979), p. 4.

represent so that he will be able to find the answer which is 5. At the enactive level he would have to use two blocks plus three blocks to get his answer Then, at the iconic level, he would need a picture of three marbles plus two marbles in order to grasp the concept of addition. An activity may include all three of these levels, or two, or even just one. The level used is determined by the needs of the learner. Some students may need the concrete representation, while others only need the pictorial representation.

Gagne', another psychologist, has developed a hierarchy of learning.[1] A student progresses from one stage to the other in the hierarchy. The first stage is known as signal learning. This stage involves a generalized emotional response. The response is essentially involuntary. Some of these responses include crying, sucking, and smiling. The next stage is called stimulus-response learning. In this stage the stimulus that causes a response is singled out. When this occurs there is a reward for the correct response, or a punishment for the incorrect response. The third stage is chaining. All that chaining consists of is

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[1]Robert Gagne', The Conditions of Learning, 2nd ed. (New York: Holt, Rinehart and Winston, Inc., 1970), cited by Linda Barron Mathematics Experiences for the Early Childhood Years, (Columbus: Charles E. Merrill Publishing Co., 1979) p. 6-8.

the sequencing of two or more stimulus response situations. The fourth stage is verbal associations. Verbal associations is characterized by a verbal sequence. No longer do we have just motor activities but a verbal representation comes into focus. An example of this is shown through memorization of the basic facts of addition. The verbal association occurs when the student has to say the basic facts of addition from memory. After verbal association, discrimination is next. At this stage, a student should be able to respond correctly to each of several stimuli that are given. In terms of mathematics, the student would have to be able to name the numbers correctly when the numerals are given in random order. The next stage is called concept learning. This is the sixth stage. The student at this stage should be able to place objects in the environment into the classes they belong to. This skill involves recognizing something that is common to those objects, such as size or shape, and thereby grouping them together. Rule learning follows concept learning. The student formulates rules based on the relationship between two or more concepts. An example of rule learning would be the ability to know which sums are even, and which are odd, when adding even numbers or odd numbers. The problem solving level is the last one according to Gagne. This level refers to rules which form higher level principles.

The student is able to solve problems within his environment. An example of this level would be a situation whereby the student is in the store and has to buy apples and oranges for a group of people. Five persons want oranges, and five want apples. The student has to be able to relate this situation and know that he has to buy five apples, and five oranges, and have a total of ten fruits.

The learning hierarchy is not necessarily equated with a particular age range. A student progresses from one stage to the other after being able to learn at that stage. Jean Piaget, on the other hand, has identified four levels of intellectual development.[1] These stages are related to age ranges. The first stage is known as the sensory-motor stage. This stage begins at birth and ends at about two years of age. The child at this stage engages in reflex actions such as crying, sucking, and grasping. These actions are ways of experiencing the environment. After the sensory motor stage, the preoperational stage begins. It begins around the age of two and continues to the age of seven. At this stage, the child is not able to reason from someone else's point of view, only his own point of view. He is unable to perform conservation tasks

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[1]Jean Piaget and Barbel Inhelder, The Psychology of the Child, (New York: Basic Books, Inc., 1969), cited by Linda Barron, Mathematics Experiences for the Early Childhood Years (Columbus: Charles E. Merrill Publishing Co., 1979), p 2-3.

such as seeing that the quantity of a set of marbles remains the same regardless of the sizes of the containers they are placed in. For example, if six marbles were placed in a large container, and six were placed in a smaller container, the child has a tendency to claim that one container has more than the other when in reality they have the same amount. The concrete operational stage begins at about seven and ends at about eleven years of age. This stage is where we see the emergence of logical thought. The student forms concepts based on his contact with concrete objects. This is especially evident in the area of mathematics where the student learns to count by having the concrete objects in front of him. He no longer is unable to see that the six marbles he starts out with are what he ends up with regardless of the size of the container they are placed in. The formal operational stage is the last stage of Piaget's developmental scheme. This stage begins approximately at age eleven or twelve and continues to age fifteen. The student is no longer dependent on concrete objects to formulate ideas. He can now formulate predictions, reason deductively, and understand hypothetical situations. The stages of learning are very important to the educator in terms of finding ways in which to educate the youngsters.

Characteristics of Educable Mentally Retarded Students

Before looking at the characteristics of the educable mentally retarded student, a definition as to who are they is needed. The American Association of Mental Deficiency defines it as the following: "Mental retardation refers to significantly subaverage general intellectual functioning existing concurrently with deficits in adaptive behavior and manifested during the developmental period."[1]

Mental retardation is divided into categories. These include educable mentally retarded, trainable mentally retarded, and the severely mentally retarded. The educable mentally retarded may have difficulty in the area of learning abstract concepts. Their appearance does not look different from that of normal children. Educable mentally retarded is categorized by an intelligence quotient range of 50 to 70 or 75. There are certain characteristics to be found in the educable mentally retarded student. One characteristic is that he is sensitive to his surroundings; this child seems to know when he is accepted or not. Researchers have advised that the child be shown alot of love and praise, instead of rejection. The important point here is the value of acceptance. According to Malinda Garton, "Acceptance is

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[1]William R. Van Osdol and Don G. Shane, An Introduction to Exceptional Children, (Iowa: Wm C. Brown Co., 1974) p. 57.

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[1]William R. Van Osdol and Don G. Shane, An Introduction to Exceptional Children, (Iowa: Wm C. Brown Co., 1974) p. 57.

important for the preservation of the child's dignity and the achievement of self realization."[1] This acceptance goes a long way in terms of the child's achievement. He would try his best to do well, knowing that someone cares about him. Another characteristic of this child is that he has a slow reaction time. He is unable to become interested in a new activity without some adjustment. There should be time for him to get adjusted to a new activity. Some time should be alloted for the student to put away materials of the previous activity before engaging in a new one. His attention span is short. With this in mind, the child has to be actively involved in the learning activity. The materials being used in the activity should be at the child's level of interest and comprehension. This is very important in enabling the child to complete the task with a feeling of satisfaction. In terms of language, the educable mentally retarded child has difficulties in the "use and comprehension of verbal and numerical symbols."[2] This child may not understand what is meant by time, or even the value of planning. He has a difficult time understanding the value of money. Along with this, he seems to be of the practical type. What

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[1]Malinda Garton, Teaching the Educable Mentally Retarded, (Springfield: Charles C. Thomas, 1964) p. 18.

[2]Ibid., p. 21.

is meant by this is that it seems as though he is unable to apply separate qualities to the solution of problems.[1] The educable mentally retarded student has little ability in terms of self-evaluation of his efforts. He has a narrow range of interests. However, according to Malinda Garton, this range of interests can be stimulated by use of audiovisual materials, field trips, and dramatizations. Other difficulties of these students include difficulty in recognizing boundaries, difficulty in distinguishing right and wrong, and difficulty in terms of emotional stability. He, however, has the ability to be loyal and acquire habits. The above mentioned characteristics are those that are often seen with educable mentally retarded students.

Characteristics of Educable Mentally Retarded Students in Arithmetic

According to Burns, the educable mentally retarded student is retarded in the knowledge of arithmetic vocabulary, ability to solve abstract and verbal problems, understanding the concept of time and sequence, and differentiating extraneous material from needed arithmetical facts.[2] These

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[1]Ibid., p. 22.

[2]Paul C. Burns, "Arithmetic Fundamentals for the Educable Mentally Retarded," American Journal of Mental Deficiency, 66 (1961) p. 58.

students are more careless at their work, make more technical errors, and use more immature processes than their normal peers. Arithmetic readiness is very important for these students. They do better in addition and subtraction than in multiplication and division. Not only this, but they do well in word problems when the situations of the problems are meaningful to them.

Another set of behaviors in regard to arithmetic was stated by Garton in her book entitled, Teaching the Educable Mentally Retarded. This author claims that the following behaviors are characteristic of the educable mentally retarded. These include: low transfer of learning, low abstract thinking ability, poor observation and comprehension of details and situations, slow absorption of facts, little initiative, and lack of ability to concentrate.[1] With these characteristics in mind, these students must be taught in such a way that they can understand, and this understanding can in turn be related to real life experiences.

Other research has indicated that these students perform up to mental age expectancy on computational skills, and functional areas such as time and money.[2] They do poorly on situations that involve concept development and

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[1]Garton, Teaching the Educable Mentally Retarded, p. 220.

[2]Frank Hewett, Education of Exceptional Learners (Boston, Allyn and Bacon, Inc. 1974), p. 367.

reasoning. These students may engage more often in elementary means of obtaining an answer, like counting of fingers, than their normal peers. Unlike their normal peers, these students do only half as much academically as their normal peers. Therefore, it is essential that what they do learn is beneficial to their everyday life.

Arithmetic That Is Needed

Based on the characteristics of the educable mentally retarded student, his arithmetic should be taught in small step sequences which are designed to produce success. This student has to obtain a level of success at his tasks or he becomes easily frustrated. Another type of activity is suggested for this student; this activity is called individualized instruction. He is given instruction based on his particular level of achievement and knowledge. He doesn't have to keep up the pace with his fellow students but he achieves based on his own ability and interest. Other activities include small group involvement. The student should also be able to work with concrete materials. It is suggested by researchers that physical activity be a part of these students' instructional experiences.[1] By physical activities,

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[1]Kenneth Lovell, The Growth of Basic Mathematical and Scientific Concepts in Children (London: University of London Press, 1961), Leo Brueckner, Foster Grossnickle and John Reckzeh, Developing Mathematical Understanding in the Upper Grades, (Philadelphia: J. G. Winston, Co., 1957), Zoltan P. Dienes, Building Up Mathematics, (London: Hutchinson Educational, 1960) cited by Austin Connolly, "Research in Mathematics Education and The Mentally Retarded", Arithmetic Teacher 20, (Oct. 1973) p. 495.

reasoning. These students may engage more often in elementary means of obtaining an answer, like counting of fingers, than their normal peers. Unlike their normal peers, these students do only half as much academically as their normal peers. Therefore, it is essential that what they do learn is beneficial to their everyday life.

Arithmetic That Is Needed

Based on the characteristics of the educable mentally retarded student, his arithmetic should be taught in small step sequences which are designed to produce success. This student has to obtain a level of success at his tasks or he becomes easily frustrated. Another type of activity is suggested for this student; this activity is called individualized instruction. He is given instruction based on his particular level of achievement and knowledge. He doesn't have to keep up the pace with his fellow students but he achieves based on his own ability and interest. Other activities include small group involvement. The student should also be able to work with concrete materials. It is suggested by researchers that physical activity be a part of these students' instructional experiences.[1] By physical activities,

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[1]Kenneth Lovell, The Growth of Basic Mathematical and Scientific Concepts in Children (London: University of London Press, 1961), Leo Brueckner, Foster Grossnickle and John Reckzeh, Developing Mathematical Understanding in the Upper Grades, (Philadelphia: J. G. Winston, Co., 1957), Zoltan P. Dienes, Building Up Mathematics, (London: Hutchinson Educational, 1960) cited by Austin Connolly, "Research in Mathematics Education and The Mentally Retarded", Arithmetic Teacher 20, (Oct. 1973) p. 495.

researchers refer to activities where the students actually move things with their hands such as concrete materials. The use of concrete action-oriented teaching aids by students is supported by Piaget, Bruechner, Grossnickle, Reckzeh, and Dienes.[1] In the arithmetic activities, reading should be kept at a minimum. Realizing the type of student one is working with, the standards of evaluation should be reasonable, based on the capabilities of the student. The diagnostic and evaluative techniques should be used frequently. This should be constantly going on so that progress may be noted and also areas of remediation. The diagnostic and evaluative techniques enable educators to make judgements as to what the student knows, doesn't know, and needs more work on.

Another suggestion is brought about concerning the arithmetic of the students. It has been suggested by Burns that these students be taught arithmetic methodically.[2] They need frequent review and maintenance exercises. The concepts that have to be taught will have to re-introduced almost every year. The review and re-introduction is needed because concepts have to be reinforced through these methods or they have a tendency to be forgotten. The levels of

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[1]Ibid., p. 495

[2]Burns, "Arithmetic Fundamentals for the Educable Mentally Retarded," p. 57.

instruction should, as much as possible, refer to specific experiences and situations to which the student can relate, and therefore get a better understanding of the concepts being taught.

There are three major jobs for the teacher of the educable mentally retarded when it comes to arithmetic. The teacher has to decide, "what is the most important to teach these students, understand the weaknesses of these students in the area of arithmetic, and decide upon methods and materials for instruction."[1] Burns, in his article feels that, based on research, it would appear that the major emphases to be placed in arithmetic are the following:

> "a strong arithmetic readiness program, use of concrete materials and manipulative aids, a variety of activities and procedures for each skill, use of grouping and individualized instruction; following an orderly system and sequence; use of considerable oral and incidental arithmetic; meaning and understanding on numbers as contrasted with mere mechanical manipulation of numbers; use of computational skills in meaningful, life-like, socialized situations; and considerable recurrence, distributive rather than concentrated.[2]

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[1]Ibid., p. 59.

[2]Ibid., p. 59.

Johnson and Mykelbust seem to view the disorders in arithmetic that the educable mentally retarded student may have as stemming from two basic problems. These problems are those in other language areas and disturbances in quantitative thinking.[1] If a student is having problems in the language area such as auditory receptive language, he may have problems in arithmetic. This may be the result of him not being able to profit from the teacher's verbal presentation of the principles. In this area, the student may not be able to understand word problem contexts which are utilized in the spoken instruction that is given by the teacher. If the student has difficulty in reading, he may have some trouble interpreting word problems. He may have difficulty in writing down his answers correctly due to poor visual motor integration.

The other aspect that was mentioned before was disturbances in quantitative thinking. Disturbances in this area may result in the student having problems in comprehending certain mathematical principles. In order for the student to acquire the skill in understanding and using quantitative relations, instruction must begin at the basic, non-verbal

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[1]D. L. Johnson and H. Myklebust, Learning Disabilities: Educational Priciples and Practices, (New York: Grune and Stratton, 1967) cited by Frank M. Hewett, Education of Exceptional Learners, (Boston: Allyn and Bacon, Inc., 1974) p. 367.

level. The principles of quantity, order, size, space, and distance must be taught. Connolly found that the developmental sequence proposed by Piaget, that was presented earlier in this study, is relevant to mentally retarded individuals also. The Piaget-type tasks, such as the conservation of quantity, are a function of the mental age of these children. According to Piaget, the notion of numbers, along with other mathematical concepts, are not learned just from teaching.[1] He feels that to a large degree, these ideas are developed by the child himself. The true understanding of the concepts involved comes about with the mental growth of the child. Piaget also feels that, if a concept is introduced prematurely, the learning that takes place is only verbal; true understanding occurs as the child grows mentally and is at the mental age to understand the concept. A child needs to understand the concept. A child needs to understand the principle of conservation of quantity before being able to develop the concept of numbers.

The work of Piaget has certain implications for the teacher engaged in the task of teaching elementary school mathematics. Some implications are as follows:

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[1] Jean Piaget, "How Children Form Mathematical Concepts". Scientific American 189 (Nov. 1953) p. 74.

1. The child's mental growth advances through qualitatively distinct stages. These stages should be looked upon when planning the curriculum.

2. Test the student to be sure he has mastered the prerequisite for that concept before introducing a new concept.

3. The pre-adolescent child makes typical errors of thinking based on his stage of mental growth. The teacher should try to understand these errors.

4. The teacher can help the child to overcome errors in his thinking by providing experiences to show the errors and ways to correct the errors.

5. The student in the pre-operational stage has a tendency to fix his attention on one variable and neglect the others. The teacher should help him overcome this by providing many situations so he may explore the influence of two or more variables.

6. Teachers should teach pairs of inverse operations in arithmetic together because a child's thinking is more flexible when it is based on the reversible operations.

7. Mental growth is encourged by experience of seeing things from many different points of view.

8. Physical action is one of the bases of learning. To learn effectively, a child must be a participant in the events.

9. There is a lag between perception and formation of a mental image. We can reinforce the developing mental image with frequent use of perceptual data.[1]

Cruickshank, in his research on mentally retarded youngsters, found results that were similar to those that were mentioned before. He found that the mentally retarded youngster was inferior to mental age normal peers in:

1. Ability to solve abstract and verbal problems.
2. Ability to solve concrete problems.
3. Their understanding of the operations to solve a problem.
4. Their ability to isolate pertinent information from a body of given data.
5. Their work habits which are characterized by carelessness and immaturity.[2]

Bower did a study on mentally retarded and normal children in which a comparison of their arithmetic competencies was made. A field test version of Key Math was used. He

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[1]Vincent Glennon and Leroy Callahan, Elementary School Mathematics: A Guide to Current Research, 3d ed. (Wash. D.C.: Association for Supervision and Curriculum Development, NEA, 1968) p. 16.

[2]W. M. Cruickshank. "A Comparative Study of Psychological Factors Involved in the Response of Mentally Retarded Children Ages Thirteen through Sixteen (Ph.D. dissertation, University of Michigan, 1946) cited by Austin Connolly, "Research in Mathematics Education and Mentally Retarded" Arithmetic Teacher 20 (Oct. 1973) p. 492.

found that the performance of the mentally retarded was inferior to that of the normal students with the same chronological age. However, when the mentally retarded were compared to normal students of similar mental age they were superior in certain areas. These areas were multiplication, division, money, time, and calendar. The normal youngsters were superior to the mentally retarded in addition, subtraction, numerical reasoning, and measurement. The results of this particular study implied that mentally retarded students perform best on computational and functional areas of arithmetic, weaknesses in areas of arithmetic requiring verbal mediation and weaknesses in work habits typified by careless computational errors, following directions, and organizing their work.[1]

There were three instructional practices or approaches suggested to be used in the arithmetic instruction for the mentally retarded.[2] According to Connolly, one approach stresses language and verbal information processing. Another is known as the manipulative and discovery approach. The third approach is the individualized

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[1]N. Bower. "A Comparison of Arithmetic Competencies by Mentally Retarded and Normal Children" (Master's Thesis, University of Missouri, 1970) cited by Austin Connolly, "Research in Mathematics Education and Mentally Retarded", p. 493.

[2]Austin Connolly. "Research in Mathematics Education and Mentally Retarded," <u>Arithmetic Teacher</u> 20 (Oct. 1973) p. 495.

instruction approach. Bereiter and Engelmann place emphasis on the approach that stresses language.[1] They feel that language is the cornerstone of math instruction. The idea that language is central to academic learning is the premise for this approach. With this approach, emphasis is placed on learning to manipulate and interpret arithmetic statements based on consistent rules. The language of arithmetic is looked at.

The second approach, manipulative and discovery approach, looks at learning through concrete action-centered learning materials. These materials provide the students with experiences so that abstractions and concepts can be understood. The third approach, individualized instructions, may incorporate some of the elements of the other two. This approach is based on frequent diagnostic assessments. The student works on a series of tasks based on his own rate. His performance on the previous task dictates what the next set of tasks will be. Based on the three approaches presented, Connolly suggests a teaching arrangement where the approaches are combined. This arrangement is as follows:

Step 1. The concepts are introduced. Explanation of the subject matter has to be in general terms. The activity is related to the child's past experience

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[1] C. Bereiter and S. Engelmann. Teaching Disadvantaged Children in the Preschool (Englewood Cliffs, N.J.: Prentice-Hall, 1966) cited by Austin Connolly, "Research in Mathematics Education and Mentally Retarded:, p. 495.

and skills.

Step 2. Manipulative materials are provided to be used in groups and by individuals.

Step 3. The child has to orally summarize the concept he has learned.

Step 4. Individual materials are provided for the child. These materials help to verify that the child has understood the concept and can apply it.[1]

Manipulative Devices

Under the umbrella of manipulative devices, we find the Cuisenaire rods. Manipulative devices are those concrete materials, which when handled by the student, enables him to attain the objective or objectives that have been identified. Since Cuisenaire rods are manipulative devices, it is appropriate that there be a focus on manipulative devices presented here. It has been noted that manipulative devices are essential to the instruction program for arithmetic. According to Van Engen, "reactions to the world of concrete objects are the foundations from which the structure of abstract ideas arises."[2]

According to certain researchers, manipulative devices seem to be good devices to use in the learning of

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[1] Connolly, "Research in Mathematics Education", p. 496.

[2] Henry Van Engen. "The Formation of Concepts", In The Learning of Mathematics: Its Theory and Practice (Washington D.C.: The National Council of Teachers of Mathematics, 1953) p. 69-98, cited by Emil Berger ed. Instructional Aids in Mathematics (Wash., D.C. Nat'l Council of Teachers of Mathematics, Inc., 1973) p. 302.

arithmetic.[1] In research conducted by Dienes, it was understood that the organism seems to wish to explore and manipulate the environment. By doing this, it is able to predict how the environment is going to respond. Adler, a psychologist, feels that physical action is one of the basics of learning. Here again, we see the need for the child to actively participate in the learning activity in order for the learning to occur. Gagné feels that instruction needs to be fundamentally based on the stimulation that is provided by objects and events. He also feels that a child can learn better, especially arithmetic, when he has an abundance of opportunities to manipulate physical objects. These objects are the stimuli, through which concepts are formed. Such learning theories as Piaget's, Bruner, and Flavell's agree that the performance of internal operation accurately is increased by the experience one gets with concrete materials.

Much of the research concerned with manipulative material is inconclusive. However, there are those who show positive results toward the use of manipulative devices. Harvin, for example, used questionnaires in his study and found that the frequency of the use of such materials appears to be a contributing factor in achievement in mathematics. Adkins and Suddeth found in their study that

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[1] Berger, Instructional Aids in Mathematics, p. 302.

their is a tendency to use more instructional materials in primary grades for motivation, influence of attitude, and the purpose of discovering relationships. Sole, on the other hand, found that the use of a variety of materials does not produce better results than a single device if both of these methods were used for the same amount of time. He feels that regardless of what materials and how many materials are used it is the amount of time spent that is important. If more time is spent, achievement is improved.

According to Bernstein, there are certain principles on the selection and use of manipulative materials. They are:

1. There should be correlation between the operations carried out with the device and those carried on in doing the mathematics with pencil and paper.

2. The aid should involve some moving parts so that it illustrates the mathematics principles that are involved.

3. The device should exploit as many senses as possible.

4. The student should have his own device along with ample time to use it.

5. Learning may proceed from using physical models to using pictures to using only symbols.

6. The use of manipulative devices should be permissive rather than mandatory to the child.

7. The device should be flexible and have many uses.[1]

[1] Allen Bernstein, "Use of Manipulative Devices in Teaching Mathematics," The Arithmetic Teacher, 10 (May 1966) p. 280.

The purpose of manipulative devices is to convey an idea. A student is aided in understanding a concept by using a manipulative device. With this purpose in mind, one has to be careful in choosing the manipulative device. Hamilton suggests some characteristics that may help in the decision. The Characteristics are:

1. The outcomes and organizations of the device must not be obscure.
2. Variety is provided.
3. The device is simple to operate.
4. The device should be easy to assemble and store.
5. The parts should not be easily lost.
6. The device should encourage communication of some sort.
7. The device should not be an end in itself.[1]

Other characteristics that should also be included are durability, attractiveness, simplicity, size, and cost of the device. The device should be able to withstand regular use by children. It should appeal to children and be designed so that it is easily manipulated by them. The cost should not provide a true embodiment of the mathematical concept to be explored along with a basis for abstraction.

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[1] E. W. Hamilton, "Manipulative Devices", The Arithmetic Teacher, 13 (October 1966): 462.

There are certain uses of manipulative devices as proposed by Reys. Manipulative devices are used to vary instructional activities, provide experiences in actual problem-solving situations, provide concrete representations of abstract ideas, provide an opportunity for students to discover relationships and formulate generalizations, provide active participation by the pupils, provide for individual differences found in pupils, and increase motivation.[1] The manipulative device, with all its uses, should be selected wisely. It should not be a substitute for teaching, but a convenient aid in the process of learning. The device should be introduced to the student in such a way that he feels comfortable using it and be able to ask questions about it, along with making errors and even correcting these errors for himself.

Many teachers have accepted the use of manipulative devices and visual aids in teaching of arithmetic. There was a study conducted which looked at the effectiveness of selected materials for teaching arithmetic. This study used three groups of first graders. It took place in Oak Park, a Michigan school, during the full school year of 1960-61. They used three sets of manipulative materials One group used a commercial set of devices called <u>Numberaid</u>.

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[1] R. E. Reys, "Consideration for Teachers Using Manipulative Materials," <u>The Arithmetic Teacher</u>, 18 (Dec. 1971): 555.

The second group used a set that was selected by the teacher, and the third group used an inexpensive set of materials. The children in the study were given achievement tests and attitude surveys. Group one's materials included a Numberaid abacus for each pupil, a demonstration model, workbooks, and guidebooks for parents. The cost per pupil was five dollars. Group two's materials consisted of ten bead factfinders, fifty plastic sticks, ten round cardboard discs, and hand-operated adding machines. The materials were used to supplement the text book. The cost per pupil was one dollar. Group three's materials consisted of text books and materials found in the first grade. These were supplemented by homemade materials of the teachers. The three groups engaged in certain aspects of mathematics such as counting, addition of whole numbers, subtraction, along with some multiplication and division. The study, which involved 654 students, found that there were no significant differences in the groups, based on arithmetic computation, reasoning, and total achievement when the mean score for I. Q. subgroups 125 and above and 99 and below were used. There were no significant differences in attitude. Based on these data, the researchers felt that expenditures for manipulative devices don't seem justified.[1]

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[1]Hardwick W. Harshman, David W. Wells, and J. Payne, "Manipulative Materials and Arithmetic Achievement in Grade I. The Arithmetic Teacher, 9 (April 1962): 191.

Another study that involved manipulative devices was conducted in the Santa Clara County. Three elementary schools were chosen, one second grade class from each school. A series of twenty lessons for thirty-five minute periods were conducted in the three modes. The modes were manipulative (M), pictorial ()), and abstract (A). The instructional period began on April 29, 1972 and ended on April 31, 1972. Their knowledge of the concept was measured by an investigator-prepared instrument. The test measured the students' ability to use multiplication concepts abstractly, the use and interpretation of numerals, operations and relation of symbols, and facility with mathematical sentences. There were no significant differences between the manipulative and pictorial groups in their ability to affect the children's concept formation in beginning multiplication.[1] This study gives no evidence to prove that concrete materials contribute more to concept formation than pictorial materials.

Finely conducted an experiment with fifty-four educable mentally retarded students. The students were presented with twenty problems in the concrete, symbolic, and pictorial modes. The concrete mode used money and

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[1]Lloyd Scott and Herman Neufeld, "Concrete Instruction in the Elementary School Mathematics: Pictorial vs Manipulative," School Science and Mathematics 76 (January 1976): 70.

actual objects where the students were tested individually. Problems involving pictures were administered as a group. the symbolic aspect was administered to the group. It was found that the educable mentally retarded performed best on the symbolic test and worst on the concrete test.[1] The experimenter wondered that the concrete test was given individually there may be a possibility that the student felt more at ease in the group than when he was singled out. Along with this, there was a lack of control of important variables such as the teacher variable, and the effects of practice.

In light of what has been previously mentioned, Smith feels that practical application should be stressed in teaching mathematics to the mentally retarded. He also feels that such instruction as money value should start with the manipulation of real things.[2] Here the need for manipulation of the concrete is emphasized.

Weber studied the effect of reinforcement of mathematics concepts. This was done with first graders. She used paper and pencil follow-up activities or manipulative materials for follow-up activities. A standardized

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[1]Carmen Finely. "Arithmetic Achievement in Menally Retarded Children: The Effects of Presenting the Problem in Different Contexts," American Journal of Mental Deficiency, 67 (Sept. 1962), p. 281-86 cited by Glennon, Elementary School Mathematics, p. 46.

[2]Frank Hewett, Education of Exceptional Learners (Boston: Allyn and Bacon, Inc., 1974) p. 369.

test was given which showed that there were no significant differences in the activities providing reinforcement. However, there is a trend that favored the groups using the manipulative materials. These students scored significantly higher on the oral test of understanding.[1]

Lucas conducted a study with first graders. His study involved attribute blocks. The purpose of the study was to look at the effects of attribute blocks on the first graders. His results were quite positive. They showed that the children illustrated a greater ability to conserve number and conceptualize addition-subtraction relations.[2] Based on what has been mentioned, there seems to be a positive feeling toward manipulative devices and their place in mathematics instruction.

Studies Involving The Cuisenaire Rods

There have been some studies done with the Cuisenaire rods. Some of the studies involved normal children, while others involved the educable mentally retarded

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[1]Audra W. Weber, "Introducing Mathematics to First Grade Children: Manipulative vs Paper and Pencil," Dissertation Abstract International, 30A (Feb. 1970): 3372-73 cited by Joseph Payne, ed., Mathematics Learning in Early Childhood (Virginia: Nat'l Council of Teachers of Mathematics, Inc., 1975), p. 53.

[2]Joseph Payne, ed. Mathematics Learning in Early Childhood, (Virginia: Nat'l Council of Teachers of Mathematics, Inc., 1975) p. 53.

children. Those studies which involved the normal children will be reviewed.

Haynes conducted a study where the effectiveness of the Cuisenaire method was compared with a conventional method of teaching multiplication. Five third grade classes were in the study. The conventional method was that method set forth in the book, The New Discovering Numbers. The Cuisenaire method was based on the contents found in Mathematics With Numbers in Color: Book A and A Teacher's Introduction to the Cuisenaire-Gattegno Method of Teaching Arithmetic. A total of 106 students were in the study. One teacher in each of the two schools taught a control and a Cuisenaire group for six weeks. A third teacher taught using only the conventional method. The students were given several tests, including the Metropolitan Achievement Tests and Primary Mental Abilities Tests. This study showed that the Cuisenaire method was no more effective in teaching multiplication to third graders than the conventional method.

Another study was conducted by Crowder. His study looked at the effectiveness of the Cuisenaire method as compared to the conventional method in teaching arithmetic to first graders. An arithmetic achievement test and an inventory were used to measure the outcome of the study. Three hypotheses were tested: 1) The experimental group's arithmetic achievement was significantly greater than the

control group's achievement, 2) Sex seems immaterial to the ability to learn arithmetic in grades one and three. 3) There would be very little difference in achievement between upper and middle groups classified by socioeconomic status. The results of the study were:

> The pupils using the Cuisenaire method learned more conventional subject matter, more mathematical concepts and skills than those that were taught using the conventional program. The average and above average students profitted most from the Cuisenaire method. Sex did not matter in terms of achievement. Socioeconomic status is an important factor in ability to learn arithmetic in first grade.[1]

It seems as though the use of objective materials helps learning of arithmetic in first grade.

Lucow conducted a study in which he used third graders. There was a Cuisenaire group and a control group in each of the eight schools. A pretest and a posttest were administered. The experimental period lasted six weeks, starting January, 1962. It was noted that the pretest revealed that the Cuisenaire group was ahead of the control group. At the end of the experiment the following conclusions were made:

> Cuisenaire method proved to be effective for teaching third grade mathematics and other methods of instruction are also effective.

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[1]Alex Blecher Crowder, Jr., "A Comparative Study of Two Methods of Teaching Arithmetic in the First Grade," Dissertation Abstracts, (Ph.D. dissertation, No. Texas Univ., 1965)

> There was evidence that the Cuisenaire method operates better in a rural setting than an urban setting.
>
> The Cuisenaire method operates better on high I. Q. and middle I. Q. in a rural setting, but not much better with low I. Q.
>
> There is a slight indication that girls take to the Cuisenaire method better than boys.[1]

The experimenter felt that the results that favored the Cuisenaire group could be based on the presence of over-aged pupils, repeaters, and deviants in the control group and the differences in mental set toward multiplication and division (The process new to control group but familiar to the Cuisenaire group). In his summary of the study, Lucow feels that the Cuisenaire method is a good one and it should be added to the third grade teacher's repetoire. He also feels that the children should be taught by whatever method they respond to. With the fact that children have individual differences, it is felt that a teacher should not limit himself to one method of instruction.

Another investigator, Passy, wanted to find out the effects of the Cuisenaire method on reasoning and computation. The program was limited to the first three grades and Kindergarten. 1,200 children at each grade level participated. They were tested in May, 1962 with the Stanford Achievement

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[1]William Lucow, "An Experiment with the Cuisenaire Method in Grade Three," American Educational Research Journal, 1 (May 1964): 166.

Test, Elementary Battery and grade scores on arithmetic reasoning and computation. There were three groups. One used the Cuisenaire method in modified elementary curriculum. The second group used a meaningful arithmetic program (not Cuisenaire). Group three was drawn from the pre-Cuisenaire third grade, in the first group. The first group were third graders. This study revealed that the third graders who used the Cuisenaire method achieved significantly less at the .05 level on the arithmetic subtest of the Stanford and Elementary Battery than the other two groups.[1]

A study was conducted to look at the effects of the Cuisenaire method on the teaching of first grade mathematics. The study began in October 1961 where nine classes of first graders in three schools were selected. Four classes in one school used the Cuisenaire method. Five classes, two in one school and three in another, used the traditional method. Both groups were given a series of three tests at the end of the school year. At the beginning of the 1962-63 school year, the students in the experimental group were assigned to three second grade classes and the controls were assigned to five second grade classes. The instruction lasted twenty-five minutes per day. At the end of the

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[1]Robert Passy, "The Effects of Cuisenaire Materials on Reasoning and Computation," The Arithmetic Teacher 10 (Nov. 1963): 440.

1962-63 school year similar tests were given. Hollis, the researcher, made these conclusions: at the end of the first year and second year, the Cuisenaire method taught traditional subject matter as well as the traditional method and the students in the Cuisenaire group acquired additional concepts and skills to those taught in the traditional programs.[1]

There was a study made in Scotland and England by Brownell. The study was looking at the effectiveness of three programs. These programs were the conventional method (experiences in grouping and use of discrete objects), the Cuisenaire method, and the Dienes method (use of multi-based blocks). The study was concerned with children who had three years of schooling. Schools were selected in Scotland and England (forty-five schools with 1,430 children). There were comparisons made between the Conventional and Cuisenaire groups in the Scottish school. The three programs were looked at in the English schools. The study revealed that the Cuisenaire method was more effective than the Conventional method in developing meaningful mathematical

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[1]Loye Hollis, "Cuisenaire-Gattegno Method with a Traditional Method, A Study to Compare the Effects of Teaching First and Second Grade Math," School Science and Mathematics 65 (Nov. 1965): 685.

abstractions. However, in the English study, the Conventional method ranked the highest in overall ranking for effectiveness in promoting conceptual maturity. The Dienes and Cuisenaire methods ranked equal in this aspect. In this study, there were uncontrolled variables. These were the differences in quality and amount of instruction, the pace of the instruction, and the objectives involved. Although the results seem to point to the Cuisenaire as being more effective, Brownell raised the question that the differences in the results may be due to the skill and enthusiasm portrayed by the teachers and not in the material itself.[1] It may be that the teacher was enthusiastic about teaching something new and this raises the problem of quality of teaching. This variable is very difficult to account for. It seems as though in the English schools, where the novelty of the Cuisenaire method no longer intact, the Conventional method seems to have been better.

An experiment was conducted in Vancouver in the Fall

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[1]Callahan, Elementary Mathematics, 4th ed., p. 15.

of 1957. It consisted of one experimental and one control group in five schools in Vancouver. Four sets of Cuisenaire rods were supplied to the experimental groups. Instruction began in October. The time period of the instruction was twenty minutes a day (ten minutes for teaching and ten for seatwork). They were given four types of tests: A Detroit beginning first grade intelligence test (September, 1957), an initial survey test in number work (January, 1958), a terminal test based on the course of the numberwork for first grade (June, 1958), and survey test for content taught with Cuisenaire materials (June, 1958). The study was conducted during the 1957-58 school year with first graders. The mean score of the Detroit test was higher for the experimental group, but it was not statistically significant. There were significant difference noted in the mean test scores on the survey test of the content taught with the Cuisenaire materials. The performance of the experimental group (Cuisenaire) was superior. This shows that the Cuisenaire method may be more effective than traditional ones with bright and slow children. The conclusions of the study are as follows:

1. There was no significant difference between the experimental and control groups in rate of learning in Grade 1 numberwork.

2. The experimental group surpassed the control group

in terms of the facility with more complex combinations of whole numbers and common fractions.

3. There were no significant differences between the groups on their performance on two problem items that required reading and reasoning.

4. In terms of gains in scores on tests of basic Grade 1 numberwork, the effectiveness of the method of instruction is independent of whether the group is bright or slow.[1]

The following year a second experiment was conducted in Vancouver. The results of this experiment were similar to the one conducted before. An analysis of variance revealed that there was a highly significant relationship between ability and achievement of the groups. There was also a difference in achievement of the groups but it was not significant. Again, the effectiveness of the method was shown to be independent of whether the group is slow or bright.

An experiment was conducted in Saskatchewan during the 1958-59 school year. It was conducted for ten months with 461 in the experimental group and 263 in the control group. A series of tests were administered to the students

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[1]Ivy Hinchliffe. "A Report on an Experiment to Evaluate the Effectiveness of Two Different Methods of Teaching Arithmetic at the Grade One Level," (Thesis, University of Manitoba, 1961) p. 46.

involved in the study. The tests were: power tests administered three times, a special test which had more problems with fractions, and the Pintner Ability Test. After the first year, it was continued for the 1959-60 school year. The experiment involved first, second, and third graders. Based on the results which showed that the Cuisenaire group got higher scores on the tests, it seems as though the method is an effective one. The study was conducted by the Saskatchewan Teachers Federation. A questionnaire was administered to the teachers involved in the study. This subjective data leaned positively toward the Cuisenaire rods. It seems as though the teachers and the students enjoyed working with the rods. After this experiment, another experiment was suggested to be conducted the following year. In 1962 the Saskatchewan Teachers Federation put together a report on the Cuisenaire method. The report consists of principles and procedures of the method. Suggested activities to be used in grades one, two, and three are also in the report. It seems as though the Saskatchewan Federation of Teachers felt that the Cuisenaire method is a good one for teaching mathematics.[1]

Karatzina and Reinshaw conducted a study in Edinburgh during the 1957-58 school year. The experimental

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[1]Saskatchewan Teachers Federation. "Cuisenaire: A Sound Approach to Teaching Mathematics," (Saskatchewan Teachers Federation, 1962) p. 3.

period was eighteen months in which there were forty boys in the group, and fourteen girls and twenty-four boys in the control group. There were three tests administered. They were the Thurstone Primary Mental Abilities Test, Moray House Picture Test, and the Schonell Diagnostic Arithmetic Test. The mean scores of the tests were close, however, there were no statistically significant differences found at the .05 level.

Hinchliffe conducted a study which involved the Cuisenaire method. The study was conducted during the 1959-60 school year in Manitoba. It lasted for ten months. There were 230 first graders in the experimental group and 189 in the control group. This study, like the others, was based on the results of tests that were administered. The experimental group was taught using the Cuisenaire method and the control group was taught using the Living Arithmetic Series. Along with the objective data, subjective data were collected through questionnaires about the advantages and disadvantages of the Cuisenaire method. The questionnaires were given to the teachers and administrators of the classes that were involved in the study. The results of the study are as follows:

1. There were greater gains in achievement of the experimental group.

2. The experimental group exhibited greater skills in

computation.

3. The measurement of ability was in favor of the experimental group.

4. The questionnaires revealed that there was a positive attitude towards the rods.

5. The questionnaires showed that the skills were developed more quickly and easily.

6. The teachers and students enjoyed using the Cuisenaire method.

7. The Cuisenaire method proved to be a challenge for the brighter students.

8. The teachers suggested using the method more.[1]

Another study was conducted using the Cuisenaire method. This study, unlike the others previously mentioned, involved preschoolers. To be exact, the study used five students, each with an average of three years. The rods were used for three months in an attempt to teach them mathematical concepts through free play. There were four girls and one boy, all of average intelligence. The method used by the researcher, Karen Beard, was one in which the children played with the rods only while they were interested in the rods. The knowledge came about through

[1]Hinchliffe, "Report on an Experiment to Evaluate the Effectiveness of Two Different Methods of Teaching Arithmetic at the Grade One Level," p. 138.

dialogue concerning the rods. These materials hastened the conceptual communication between the children themselves and between the teacher and the children. The researcher made suggestions and invented games in which the students engaged in. They were tested before and after the treatment period. The tests that were used included an awareness test (made up by the researcher), and the Incomplete Man Intelligence Test. The awareness test contained questions on geometry, naming of colors, counting, reading numbers, and relationships among colored lengths of paper.

The students went through four stages while working with the Cuisenaire rods. These stages were: free play (manipulating the rods any way the students wants to explore the rods), free play with directed activities in which the relationships were observed, free play with mathematical notation, and free play with number introduction and written work.[1] Free play is needed so that an atmosphere of creativity and curiosity is established. The purpose of free play are:

1. To present a situation from which a student can unconsciously learn math.

2. To discover what can be done with the rods and see

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[1]Dagny Karen Beard, "An Intensive Study of the Development of Mathematical Concepts Through the Cuisenaire Method in Three Year Olds," (Thesis, Southern Connecticut State College, 1964) p. 31.

what pattern can be made.

3. To provide the teacher with an opportunity to see during the play, what concepts may be forming in the student's mind based on what he does with the rods.

4. For aesthetic pleasure and enjoyment.[1]

In the three months where sessions were conducted three times a week for a period of forty-five minutes to an hour there were some positive conclusions made. Beard feels that the rods were constructive in helping the children to visualize the concept of number, and such operations as addition and subtraction. It was observed that when the student became familiar with the processes involved, he no longer needed the rods and dropped them voluntarily. The students learned by themselves through the dialogue with the rods, observations of others, the involvement in games and discussion about the rods. The results of the study proved the contention of Cuisenaire developers that algebra precedes arithmetic.[2] In working with the rods, the students look at relationships of the rods (for example, one red rod is twice as long as the white rod). This can be written as W+W=R. Algebra is illustrated here by the use of letters. Later, arithmetic comes into play with the

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[1]Ibid., p. 31.

[2]Ibid., p. 55.

use of letters. Later, arithmetic comes into play with the use of numbers.

The studies concerning the Cuisenaire rods that were mentioned before involved the normal children found in the regular classroom. Callahan also conducted a study. He used retarded children in his study. The study started in February, 1966 and lasted for nine weeks. It involved a class of mentally retarded students (nine) whose age range was between seven and ten, and whose I.Q. score range was less than eighty and as low as fifty-seven. The first few classes were devoted to free play of the rods and getting acquainted with the physical properties of the rods. Then they engaged in identifying colors and assigning letter names to the colors. They were taught to build staircases with the rods and to make equations. This introductory period lasted three weeks. The fourth week was spent on developing the basic understanding of addition, subtraction and multiplication facts from one to five. At the end of the fourth week, a test was adminstered. The next five weeks were spent on establishing math facts of numbers six to ten. At the end of the experimental period, the rods were evaluated based on the structure of the child's ability. There were nine important points that came about by this study. These are:

1. The rods satisfied all areas of the Boston School Document of the course of study for the special classes at

the elementary level. These areas are development of an understanding of number concepts, teach the fundamental processes, application of the four processes to a problem, development of an understanding of measurement, and good work habits.

2. The concepts appear to remain with the student after the rods are removed.

3. The number facts were learned from concrete situations.

4. The rods encouraged the ability to recognize reverse operations (subtraction is the reverse of addition).

5. The commutative property is illustrated in a phyical situation by making trains.

6. The rods provide an opportunity for more advanced math than the coventional method.

7. The results achieved by the rods are better than achievement without the rods.

8. The children do not appear frustrated when manipulating the rods.

9. The rods enabled the students to make discoveries and retain information.[1]

The study seems to have results that are positive in terms

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[1]John Callahan and Ruth Jacobson, "An Experiment with Retarded Children and Cuisenaire Rods," The Arithmetic Teacher, 14 (January 1967) p. 13.

of the effectiveness of the materials. It seems as though these materials work pretty well with the educable mentally retarded.

Suggested Mathematics for the Educable Mentally Retarded

The type of math that was suggested for these students was a comprehensive model which stressed verbal information processing. The model would also stress conceptual development and numerical reasoning. This type of model was suggested by Crawley and Vitello.

When teachers were asked what type of mathematics instruction to be used with EMR students, many of them suggested that these students be given more time in working through their arithmetic assignments in the regular math program. Along with this, the notion of more use of concrete materials is generally accepted by teachers of EMR students.

Kirk and Johnson feel that the context to be taught to these students should be chosen on the basis of the following two principles: 1) content must include the knowledge, skills, and concepts that will be of the most value to these students later in life and 2) methods that are used should be determined by the special disabilities or abilities of these children.[1]

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[1]S. Kirk and O. Johnson. Educating the Retarded Child (Boston: Houghton Mifflin Co. 1951) cited by Callahan Elementary School Mathematics, p. 71

Crawley and Goodman stressed the use of manipulative and pictorial devices in working with educable mentally retarded students. These devices seem to help in the learning situation of these students. There was a demonstration program illustrated by Crawley and Goodman. The results showed an improvement in verbal problem-solving and understanding of the principles involved.

Goodstein, Bessant, Thibodeau, Vitello, and Vlahakos conducted a study which looked at the verb al problem-solving ability of the educable mentally retarded student. They found that the use of pictorial aids helped these students obtain a greater degree of performance in that area.

Others suggest the use of programmed instruction. Some of the benefits that were suggested by the use of programmed instruction were a reduction of time required to obtain a skill and a reduction in negativism and hostility toward the method of instruction. However, there is no clear and consistent advantage of this method over others. As a matter of fact, some researchers such as Smith and Quanhebush found that the students seem to need additional reinforcement besides that given by the machines. Most of the students needed approval by the teacher.

It is a common notion among educators that learning should be meaningful. The teacher of an EMR student has to be able to teach what is essential for the students to know and yet teach it in a way that there are developmental and

practice activities that lead to understanding and application. It has been suggested by Callahan that the teacher of these students pay attention to the principles of development that govern the child's mode of thinking, the principles of cognitive adaptation (conservation, equivalence, and flexibility), and the factors that are related to visual perception.[1]

In light of what has been mentioned concerning what type of mathematics to teach EMR students, it seems as though the majority of what is being taught is the same as what is being taught in the regular class. According to Cawley, "There isn't a single comprehensive arithmetic program that has been developed, tested and validated for use throughout school age range for the mentally handicapped children."[2] He also feels that until such a program is established the educator will not know whether the low achievement is the result of poor instruction, the curriculum, or the limitations of the individual. One of the greatest limitations in terms of searching for a good program for the mentally handicapped (another term for EMR) is the notion that these students are concrete learners. This notion has led to a

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[1]Callahan, Elementary School Mathematics, p. 72

[2]John F. Cawley, "Teaching Arithmetic to Mentally Handicapped Children," Strategies for Teaching Exceptional Children, (Denver: Love Publishing Co., 1972) p. 250.

the present generation, along with future generations, may be able to live comfortably in the world.

CHAPTER III

DESIGN

The study looked at the following hypothesis: There will be a significant difference in the mean of the posttest scores of the students using the Cuisenaire rods as compared to the mean of the posttest scores of those students using the traditional method when adjusted for previous mathematics knowledge and intelligence.

SOURCE

The study involved two primary educable mentally retarded classes. One was at the Ulla Muller Elementary School and the other one was at the Peace Corps Elementary School. Eighteen educable mentally retarded students were involved, ten in the experimental group and eight in the control group. The age of the students ranged from nine to eleven years.

PROCEDURE FOR COLLECTING DATA

The study was quasi-experimental in design. Both the experimental group and the control group were tested prior to the treatment period. At the end of the experimental period, a posttest was administered to both groups. The

test that was used was the Stanford Achievement Test Primary Level II. Only the mathematics computation test of the battery was used. The test items were randomly split in half along with the time for administration of the test. One-half of the test served as the pretest and the other half was the posttest. Test time for both tests was fifteen minutes.

Both the control and experimental groups remained in their self-contained classrooms. The researcher taught both the experimental and control groups during the three week period. Throughout the experimental period of three weeks, the experimental group worked with Cuisenaire rods to solve addition and subtraction problems involving whole numbers. The control group used the Mathematics Around Us textbook for grade two, Publisher - Scott, Foresman and Company, along with workbook exercises and drill exercicises placed on the chalkboard. Both groups were taught each day for a period of three weeks for approximately sixty minutes each day. The mathematics problems consisted of one and two digit addition and subtraction problems.

ANALYSIS OF DATA

The data was analyzed by the use of the analysis of covariance. The I. Q. and pretest scores were used as

the covariates. The posttest scores were the dependent variables and the treatment was the independent variable. There was a comparison made of the pretest and posttest scores of both groups to see if any gains in achievement were made. The T Test was used to analyze this.

CHAPTER IV

THE FINDINGS

The research was conducted over a period of three weeks where two groups of educable mentally retarded students were taught the above mentioned concepts. One group, the experimental group, used the Cuisenaire rods while the other group, the control group, used the traditional method. Both groups were taught by the researcher for approximately one hour, five days a week. Each group was given a pretest at the beginning of the three week period, and a posttest at the end of the period. The posttest scores served as the dependent variable, and the treatment received by each group served as the independent variable. The criterion variable is the posttest, and the control variables are the pretest and the I. Q. scores.

Table I presents descriptive data on pre- and posttest and I. Q. scores of the two groups. It was shown by the analysis of covariance that the null hypothesis cannot be rejected at the .05 level of significance (Table II). There were no significant differences in the means on the posttest scores at the .05 level of significance.

The T Test that was used for both groups showed that the difference between the means of the pre- and post-test

TABLE I

MEANS OF PRETEST AND POSTTEST FOR TWO GROUPS OF EMR STUDENTS IN MATHEMATICS

Method	n	Pretest Means	Posttest Means	I.Q. Means
Traditional	8	51.38	74.25	59.75
Cuisenaire	10	52.90	72.30	64.30
Total	18	104.28	146.55	124.05

TABLE II

ANALYSIS OF COVARIANCE FOR ACHIEVEMENT DIFFERENCES BETWEEN TWO EXPERIMENTAL MATHEMATICS GROUPS CONTROLLING FOR PRIOR MATHEMATICS ACHIEVEMENT AND INTELLIGENCE

Source of Variation	Degrees of Freedon	Sum of Squares	Residuals Mean Square
Between	1	56.6	56.6
Within	14	2210.0	157.86
Total	15	2266.6	

$$F = \frac{\text{mean square (between)}}{\text{mean square (within)}} = \frac{56.6}{157.86} = .359$$

CHAPTER V

DISCUSSION AND RECOMMENDATIONS

This study attempted to show the effectiveness of the Cuisenaire Rods with educable mentally retarded students. It was found that there was no significant difference in the two methods employed. There was achievement in both groups but there was no significant difference of one method over the other. Therefore, the method employed did not make much of a difference in the achievement.

Throughout the three week period, there are certain observations made by the researcher that needs to be noted here. In the review of the literature, certain characteristics of the educable mentally retarded were mentioned. Some of these characteristics were observed by the researcher. These characteristics are: 1) The use of fingers in counting, 2) They made a lot of mistakes, 3) Repetition was needed in order for the concept to be maintained, 4) They have a tendency to forget easily and, 5) They were slow in absorbing facts.

Based on the findings of the study, the following recommendations are submitted:

1. A study similar to this one should be conducted for a longer period of time. A suggested period of two

APPENDIX A

SCORES OF TWO GROUPS (CUISENAIRE AND TRADITIONAL) ON A PRETEST (X_1), A POSTTEST (Y), AND I. Q. (X_2)

SUBJECTS	Y	X_1	X_2
TRADITIONAL			
1	59	41	70
2	65	41	64
3	88	53	49
4	76	41	41
5	100	88	71
6	71	71	72
7	76	47	59
8	59	29	46
TOTAL	594	411	478
CUISENAIRE			
9	82	59	68
10	94	53	55
11	71	47	67
12	65	59	55
13	47	41	54
14	76	82	66
15	65	29	74
16	59	59	69
17	76	41	68
18	88	59	67
TOTAL	723	529	643
GRAND TOTAL	1,317	940	1,121

APPENDIX B

PRETEST

Samples

A. 3+1= □ 3 4 5 2 NH O ● O O O	B. □ -2=1 5 4 2 3 NH O O O O O	C. 2 3 +1 3 3 7 5 4 NH O O O O O O
1. 2 +7 8 9 10 7 NH O O O O O O	2. 9 +6 17 14 16 13 NH O O O O O	3. 8 +6 14 3 1 2 NH O O O O O
4. 15 -9 6 14 5 7 NH O O O O O	5. 7 8 +8 22 23 24 21 NH O O O O O	6. 13 -6 13 10 7 5 NH O O O O O
7. 109 -43 143 66 103 43 NH O O O O O	8. 11-4=□ 6 8 7 15 NH O O O O O	9. □+3=11 9 14 7 8 NH O O O O O
10. □-7=6 7 13 1 11 NH O O O O O	11. □ -9=9 17 3 0 18 NH O O O O O	12. 83 +46 139 21 119 129 NH O O O O O
13. 127 -63 64 144 44 180 NH O O O O O	14. 108 -54 104 17 154 54 NH O O O O O	15. 98+9=□ 105 107 26 917 NH O O O O O

Note: The meaning of the symbols below:
> is greater than
< is less than
= is equal to

Samples

A. 5 O 3+1 B. 4 O 2+3 C. 2+4 O 3+3	> O ● O	< O O O	= O O ●
16. 7+8+6 O 8+5+7	> O	< O	= O
17. (7+6)+8 O 6+(6+8)	> O	< O	= O

APPENDIX C

POST-TEST

Samples

A. 3+1=□	3 4 5 2 NH
B. □−2=1	5 4 2 3 NH
C. 2 / 3 / +1	3 7 5 4 NH
1. 4 / +5	8 9 7 10 NH
2. 8 / +5	12 13 14 11 NH
3. 7 / −5	1 3 2 12 NH
4. 9 / 7 / +8	21 23 22 25 NH
5. 43 / +63	116 96 16 NH 106
6. 79 / −52	26 23 25 28 NH
7. 3+4=□	6 8 7 9 NH
8. 2+□=8	10 5 6 7 NH
9. □+5=12	NH 7 17 8 6
10. 10−□=4	5 7 6 4 NH
11. 63 / +55	19 128 118 108 NH
12. 175 / +73	248 148 1148 247 NH
13. 128 / −42	160 86 126 26 NH
14. 54+7=□	59 16 51 61 NH

Note the meaning of the symbols below:
> is greater than
< is less than
= is equal to

	>	<	=
Samples			
A. 5 o 3+1	>o	<o	=o
B. 4 o 2 3	>o	<o	=o
C. 2+4 o 3+3	>o	<o	=o
15. 5+4 o 4+5	>o	<o	=o
16. 11−8 o 12−7	>o	<o	=o
17. (5+9) 3 o 5+(4+9)	>o	<o	=o

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